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Fast Robots

Sensors for Mobile Robots

- **Contact Sensors:** Bumpers
- **Internal Sensors**
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- **Proximity Sensors**
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range finders (triangulation, tof, phase)
  - Infrared (intensity)
- **Visual Sensors:** Cameras
- **Satellite-based sensors:** GPS

Probabilistic Sensor Model

$$p(z|x)$$

Bayes Filter

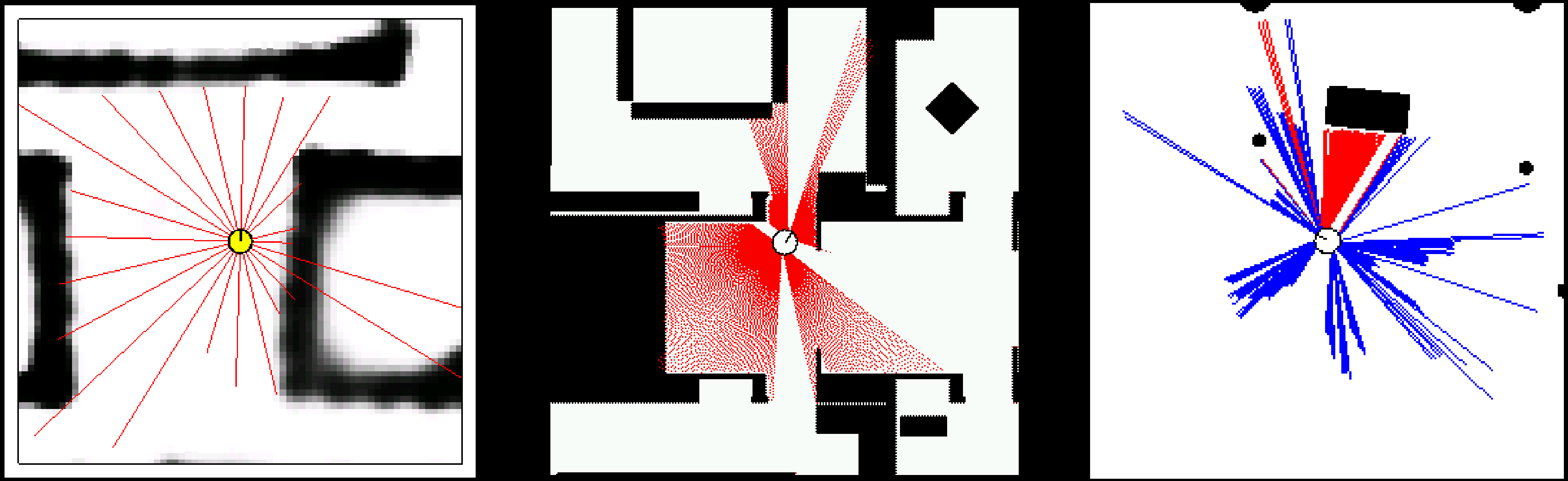
1. **Algorithm Bayes_Filter** $(bel(x_{t-1}), u_t, z_t)$:
2. for all x_t do
3. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$ [Prediction Step]
4. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$ [Update/Measurement Step]
5. endfor
6. return $bel(x_t)$

Measurement Probability / Sensor Model

Sensor Model

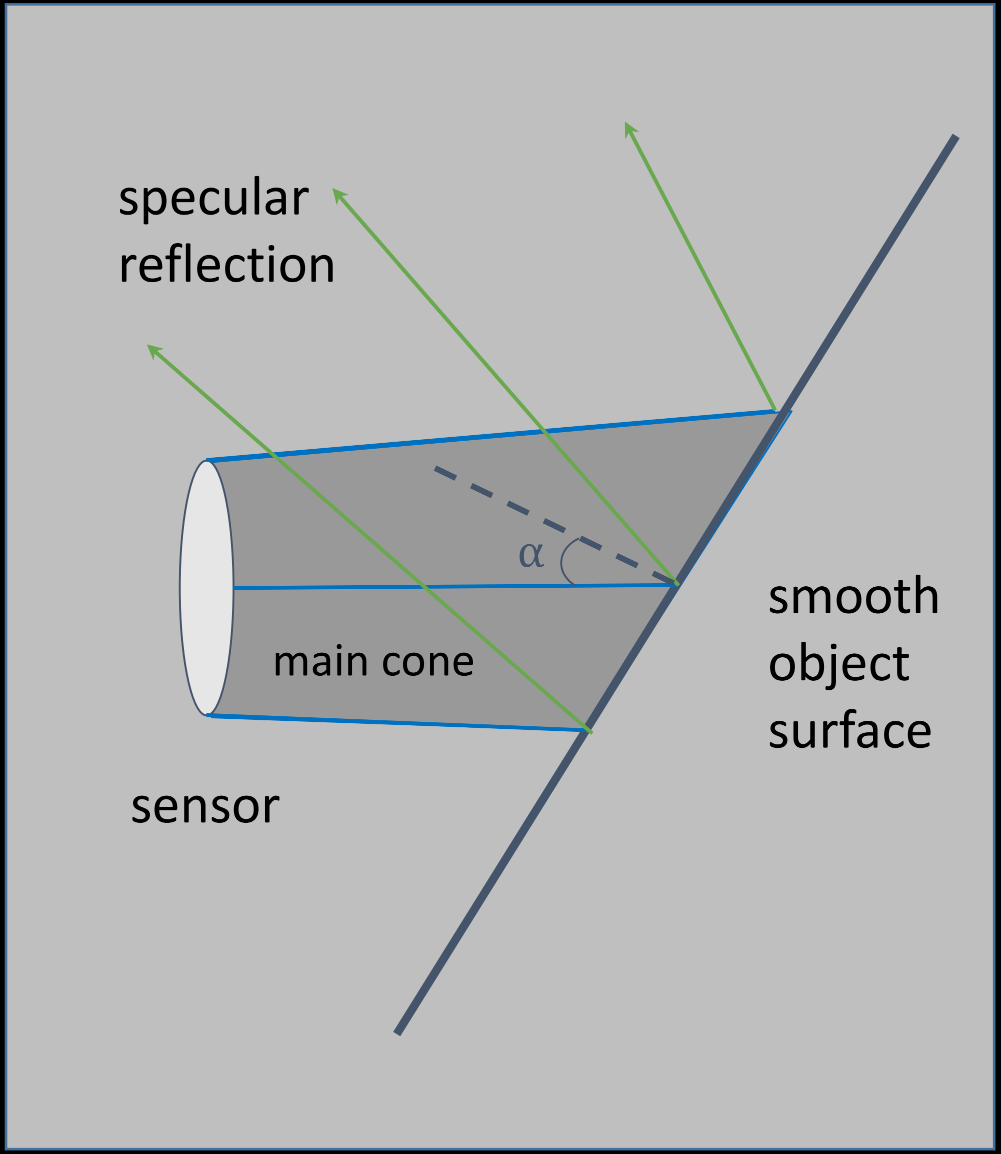
- Probabilistic robotics explicit models the noise in sensor measurements
- Sensor Model discussions are based on range sensors but can be easily transferred to other types of sensors
- The central task is to determine $p(z|x)$, i.e., the probability of a measurement z given that the robot is at position x

Where do the probabilities come from?



Range Sensor

- A smooth surface acts like a mirror (specular) to a range sensor such as an ultrasonic range sensor
- This can be problematic when rays hit at an angle; can lead to larger measurements than true values
- The inability to reliably measure range to nearby objects is often paraphrased at sensor noise



Range Sensor Inaccuracies

- Larger readings occur due to:
  - Surface material
  - Angle between surface normal and direction of sensor cone
  - Range of the surface
  - Width of the sensor cone of measurement
  - Sensitivity of the sensor cone
- Shorter readings may be caused due to
  - crosstalk between different sensors
  - unmodeled objects in the proximity of the robot, such as people

Probabilistic Sensor Model

- A sensor cannot be modelled accurately primarily due to complexity of the physical phenomena
- The response characteristics of a sensor depends on variables we prefer not make explicit in a probabilistic robotics algorithm (such as the surface properties of a material)
- Instead, it accommodates inaccuracies of sensor models in the stochastic aspects by modelling the measurement process as a conditional distribution density $p(z|x)$ instead of a deterministic function $z = f(x)$

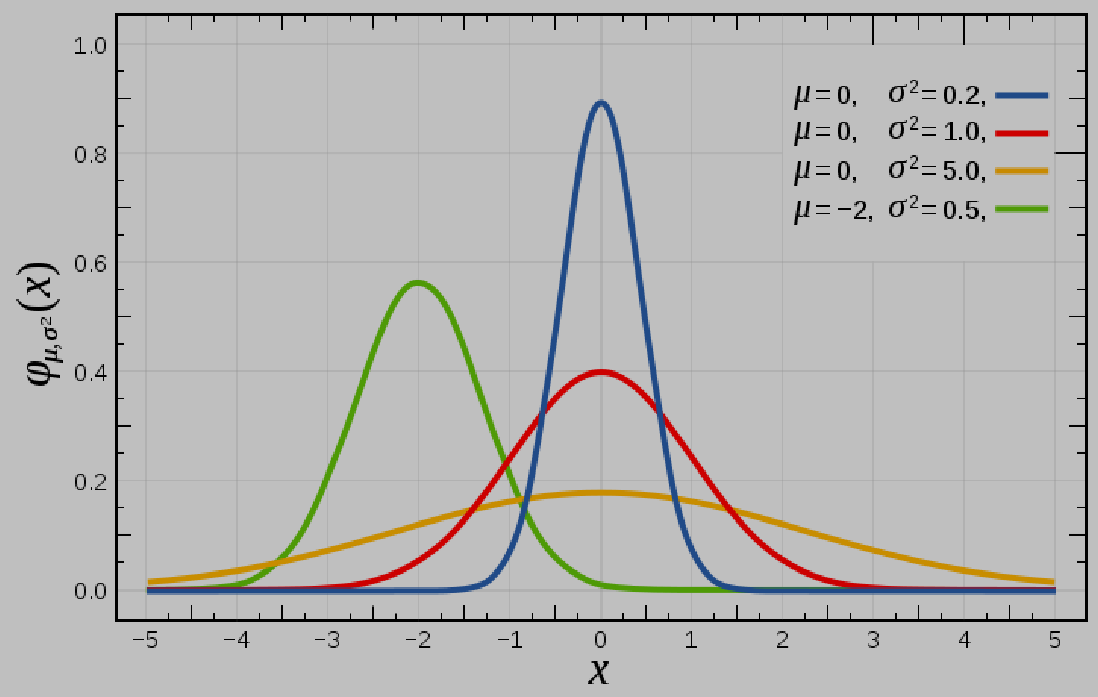
Types of sensor models

- Beam Model
- Likelihood Model
- Feature Based Model

Quick Detour: Gaussian Distribution

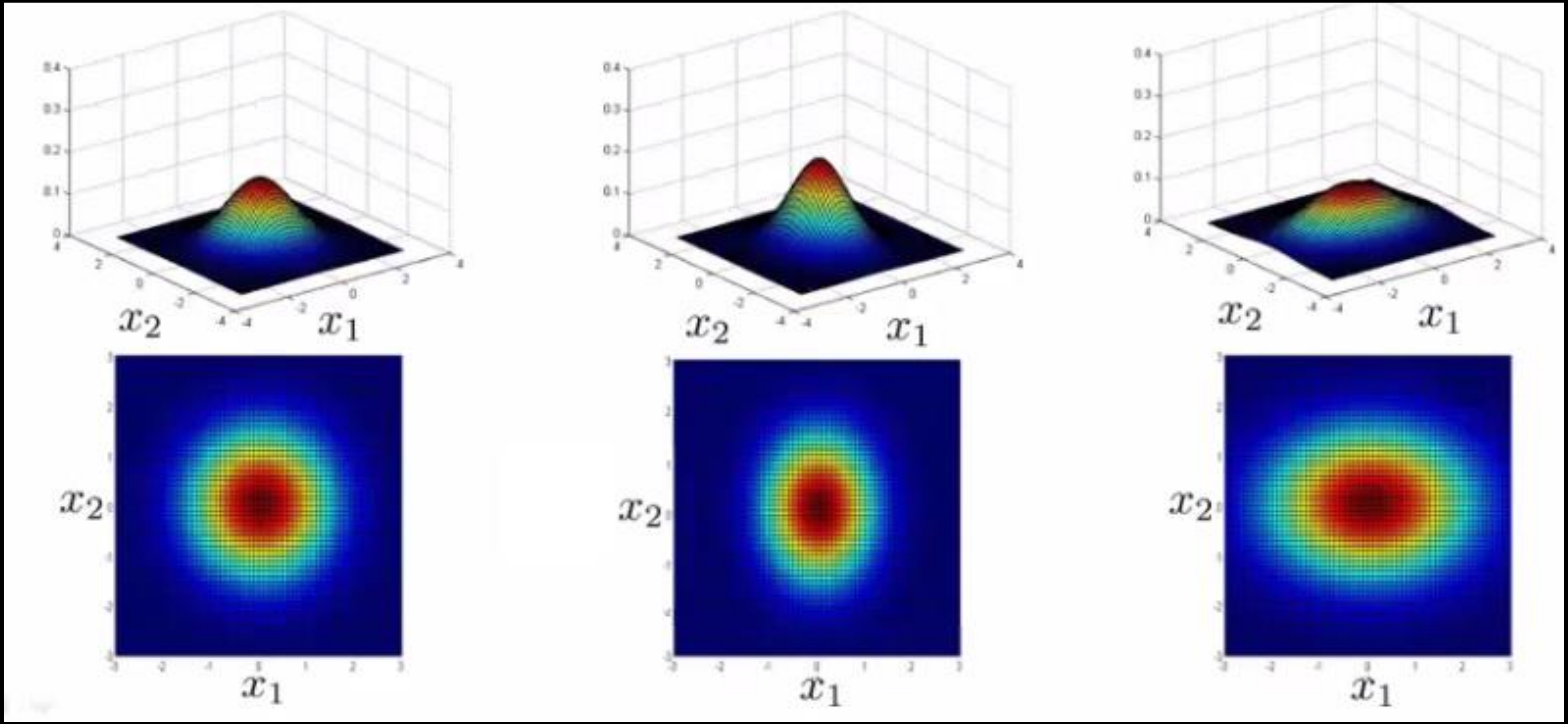
- Also known as the normal distribution or the "bell curve"
- Defined by two parameters:
  - mean μ
  - standard deviation σ
- Given a data point x, we can get how "*probable*" (relative likelihood) the value is in the gaussian distribution for a given mean and standard deviation using the probability density function $f(x \mid \mu, \sigma^2)$ or $\mathcal{N}(\mu, \sigma^2)$
- Can be defined for multidimensional data

1D Gaussian Probability Density Function



$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Quick Detour: Gaussian Distribution



Beam Model

Beam Model of Range Finders

- Let there be K individual measurement values within a measurement z_t

$$z_t = \{z_t^1, z_t^2 \dots, z_t^K\}$$

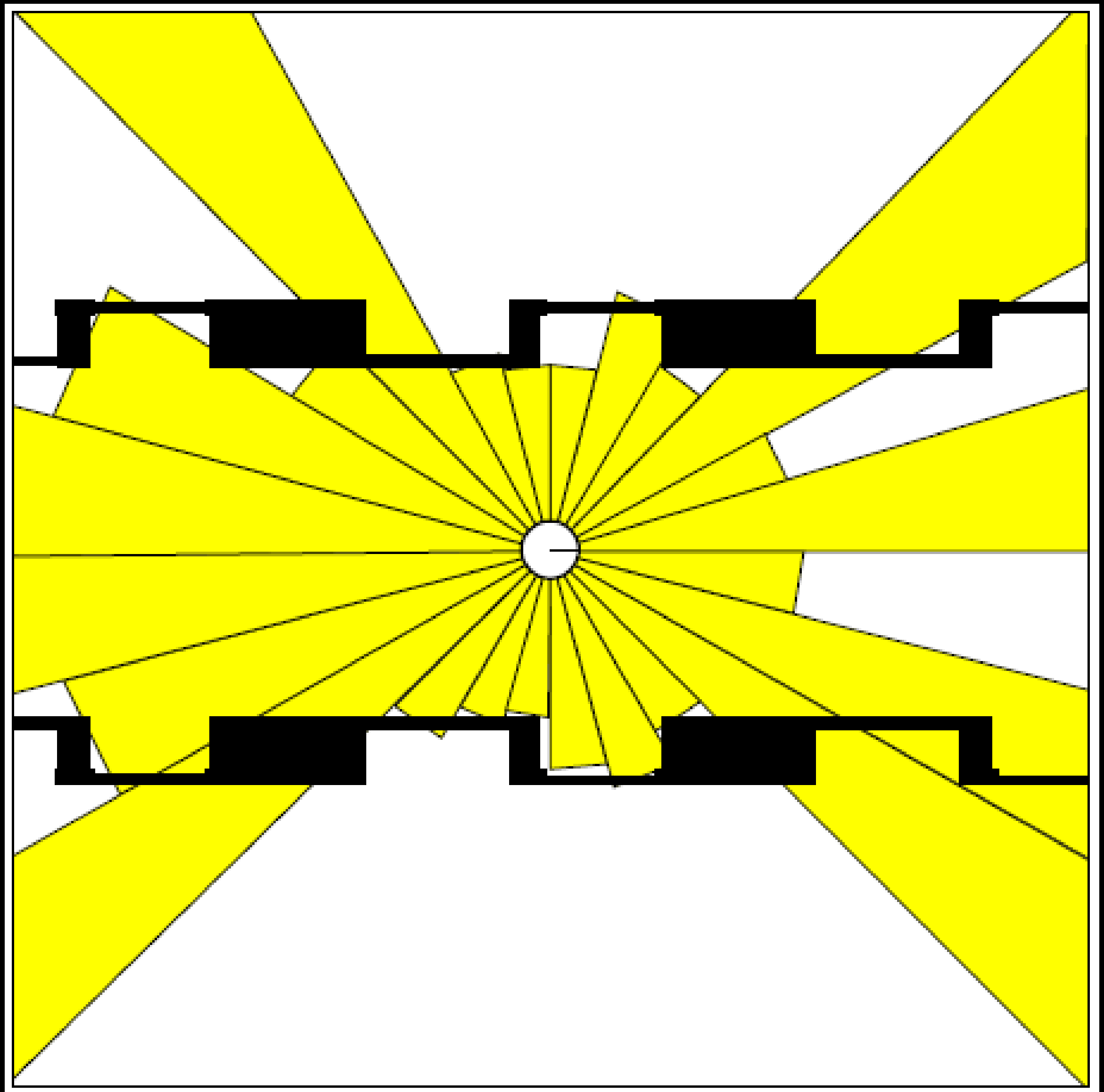
- Individual measurements are independent given the robot state

$$p(z_t | x_t, m) = \prod_{k=1}^{K} p(z_t^k | x_t, m)$$

- Dependencies exist due to a range of factors: people, error in the map model *m*, approximations in the posterior, etc

NOTE: Technically sensor measurements are caused by physical objects in the real world and not the map, however, traditionally sensor models are conditioned on the map *m*

Typical Measurement Errors of an Range Measurements



Typical Measurement Errors of an Range Measurements

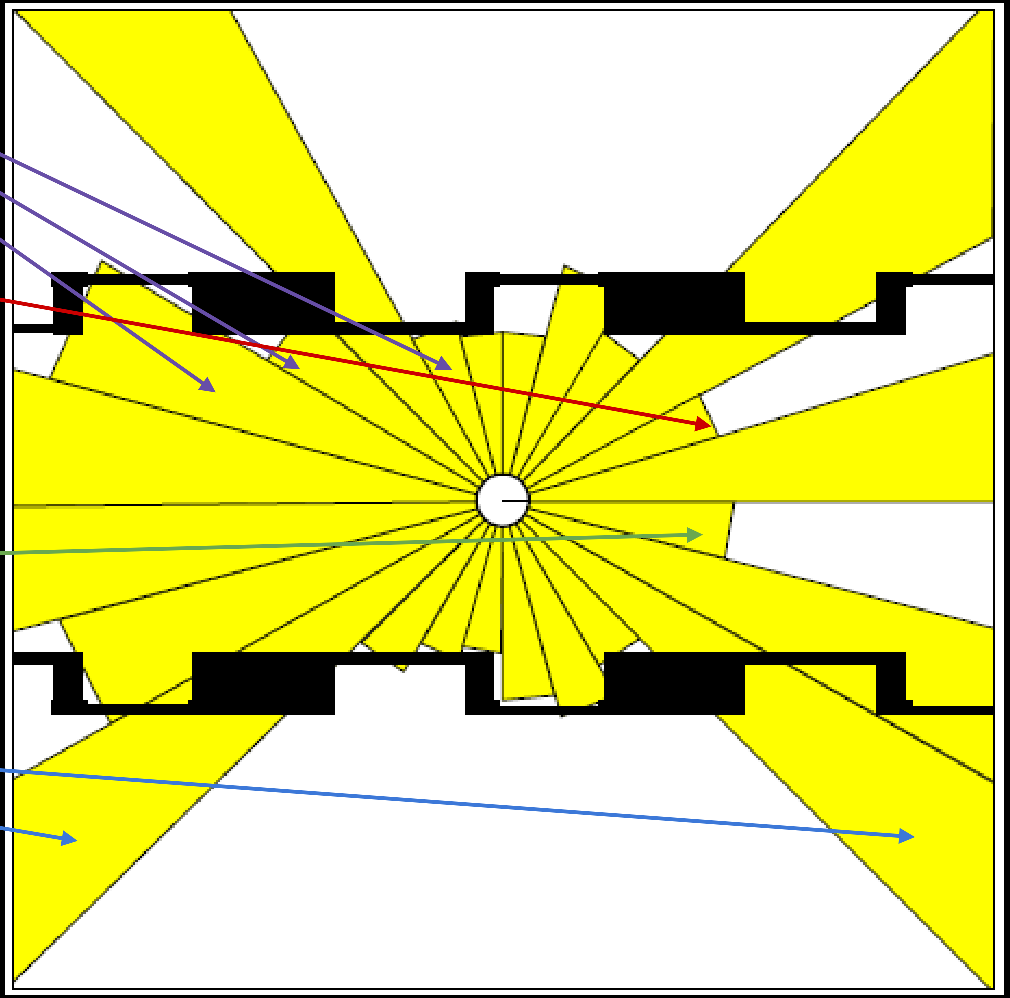
1. Correct Range Measurements:

   Beams reflected by obstacles

2. Unexpected Objects: Beams reflected by persons / caused by crosstalk

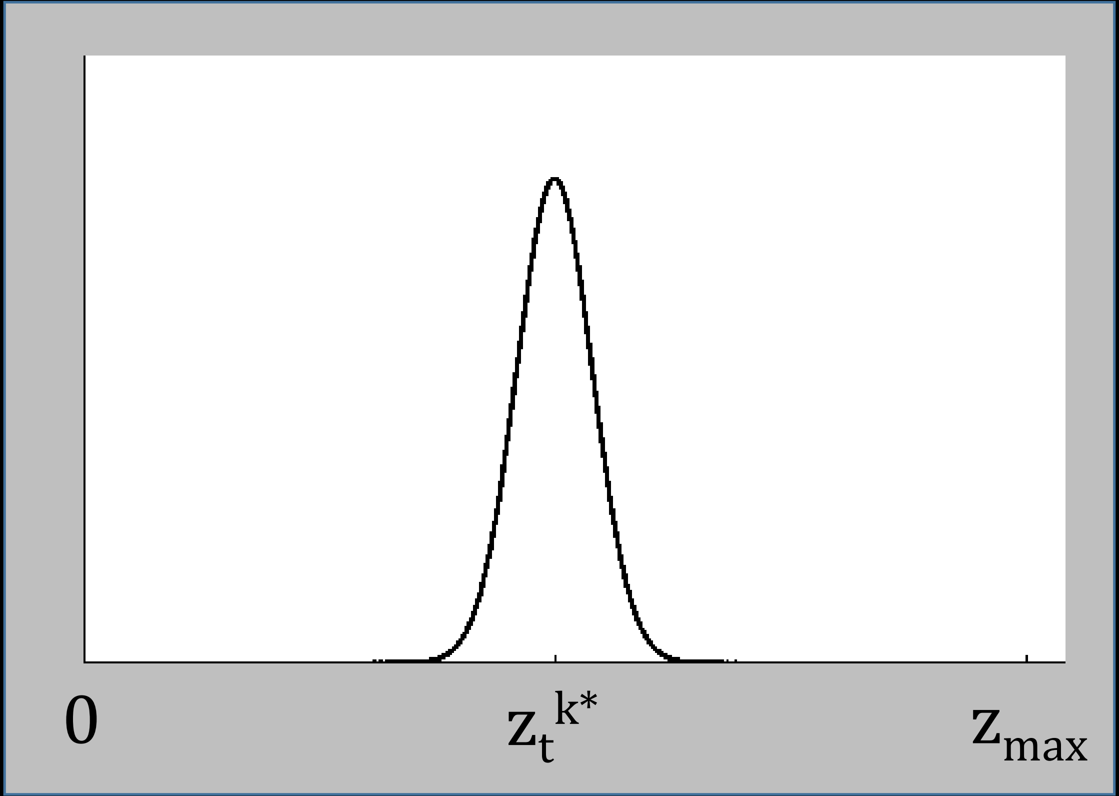
3. Failures

4. Random measurements



1. Correct Range Measurements

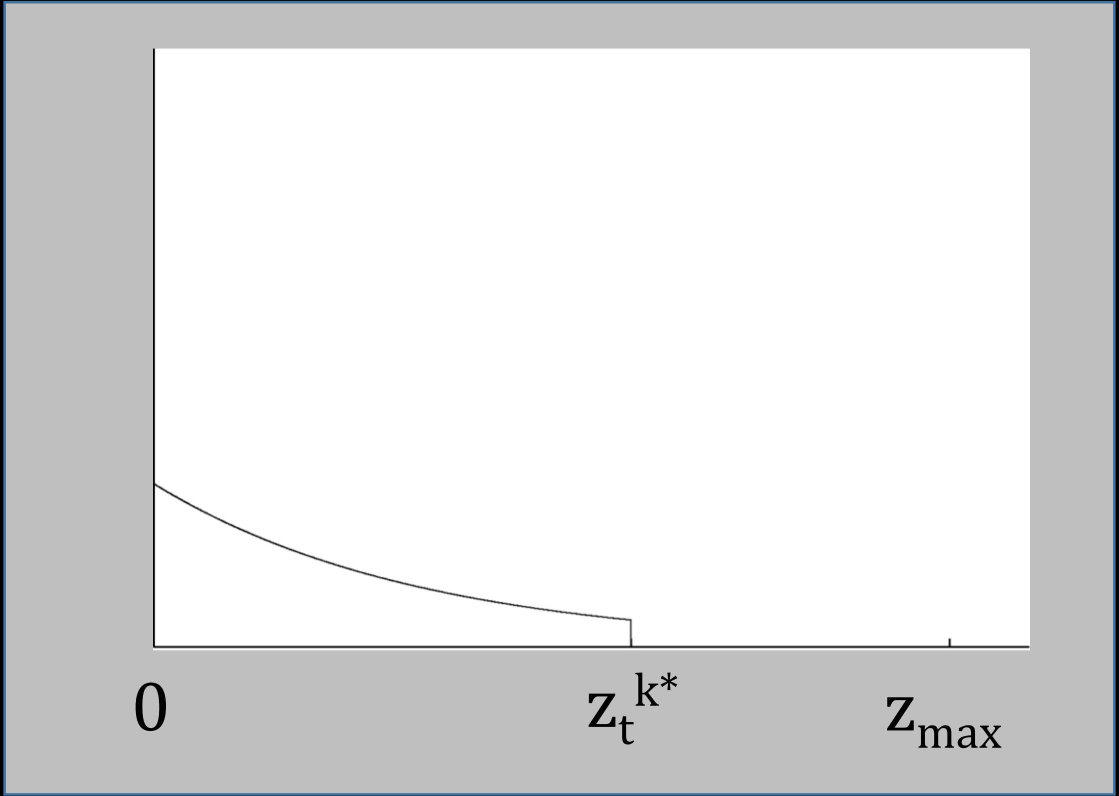
- Let z_t^{k*} denote the "true" range of the object measured by z_t^k
- Given a location-based map, the true value is usually estimated by *ray casting*
- The value returned by the sensor is subject to error, due to limited resolution, atmospheric effects, etc
- Measurement noise modelled by a narrow Gaussian p_{hit} with mean z_t^{k*} and standard deviation σ_{hit}



$$p_{hit}(z_t^k | x_t, m) = \begin{cases} \eta \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{hit}) & if\ 0 \leq z_t^k \leq z_{max} \\ 0 & otherwise \end{cases}$$

2. Unexpected Objects

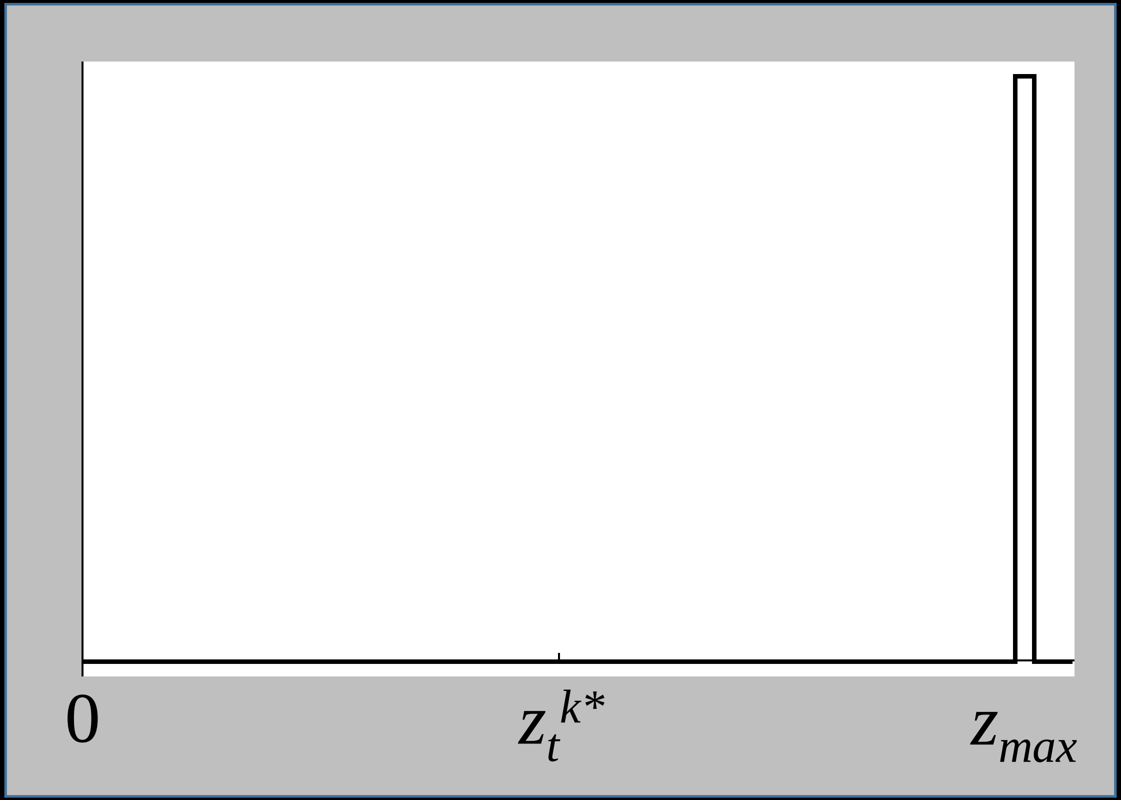
- Real world can be dynamic
- Objects not contained in the map can cause range finders to produce surprisingly short ranges (ex: people)
- Either
  - treat them as part of the state vector and estimate their location (or)
  - treat them as noise
- The likelihood of sensing unexpected objects decreases with range
- Model using an exponential distribution p_{short}



$$p_{short}(z_t^k | x_t, m) = \begin{cases} \eta \, \lambda_{short} \, e^{-\lambda_{short} \, z_t^k} & if \; 0 \le z_t^k \le z_t^{k*} \\ 0 & otherwise \end{cases}$$

3. Failures

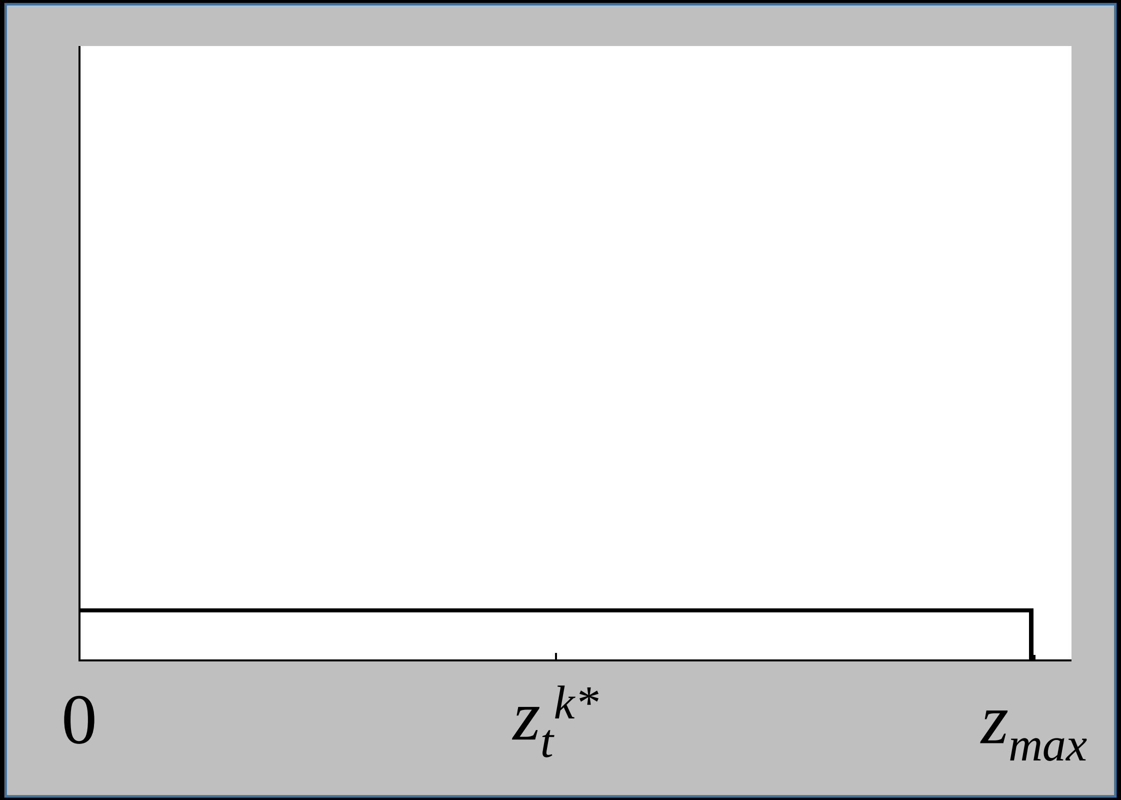
- Obstacles might be missed altogether
  - in sonar sensors as a result of specular reflections
  - in laser sensors when sensing black surfaces
- The result is a max-range measurement z_{max}
- Model as a point-mass distribution p_{max}



$$p_{max}(z_t^k | x_t, m) = I(z = z_{max}) = \begin{cases} 1 & if\ z = z_{max} \\ 0 & otherwise \end{cases}$$

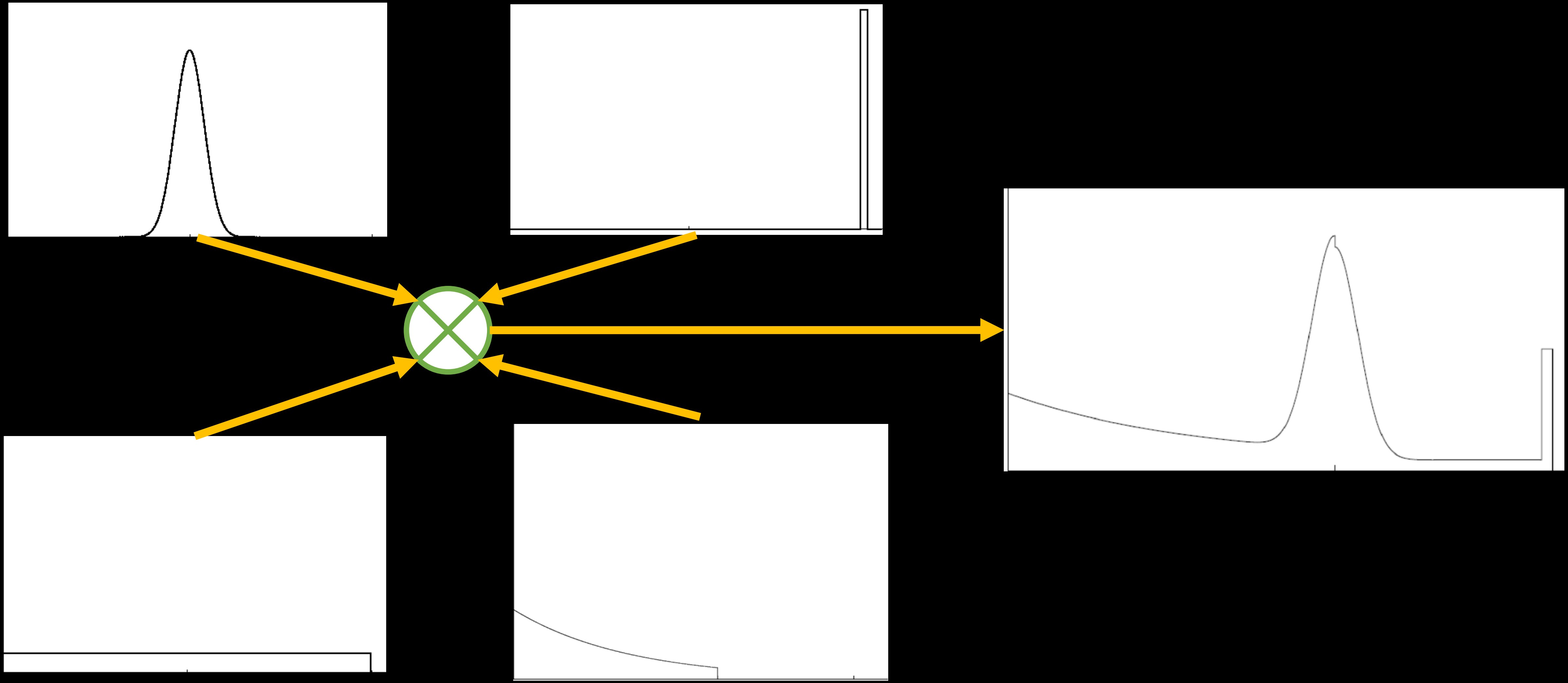
4. Random Measurements

- Range finders can occasionally produce entirely unexplainable measurements due to phantom readings, cross-talks, etc
- Modelled as a uniform distribution p_{rand} spread over the measurement range



$$p_{rand}(z_t^k | x_t, m) = \begin{cases} \frac{1}{z_{max}} & if\ 0 \leq z_t^k \leq z_{max} \\ 0 & otherwise \end{cases}$$

Beam Range Model as a Mixture Density

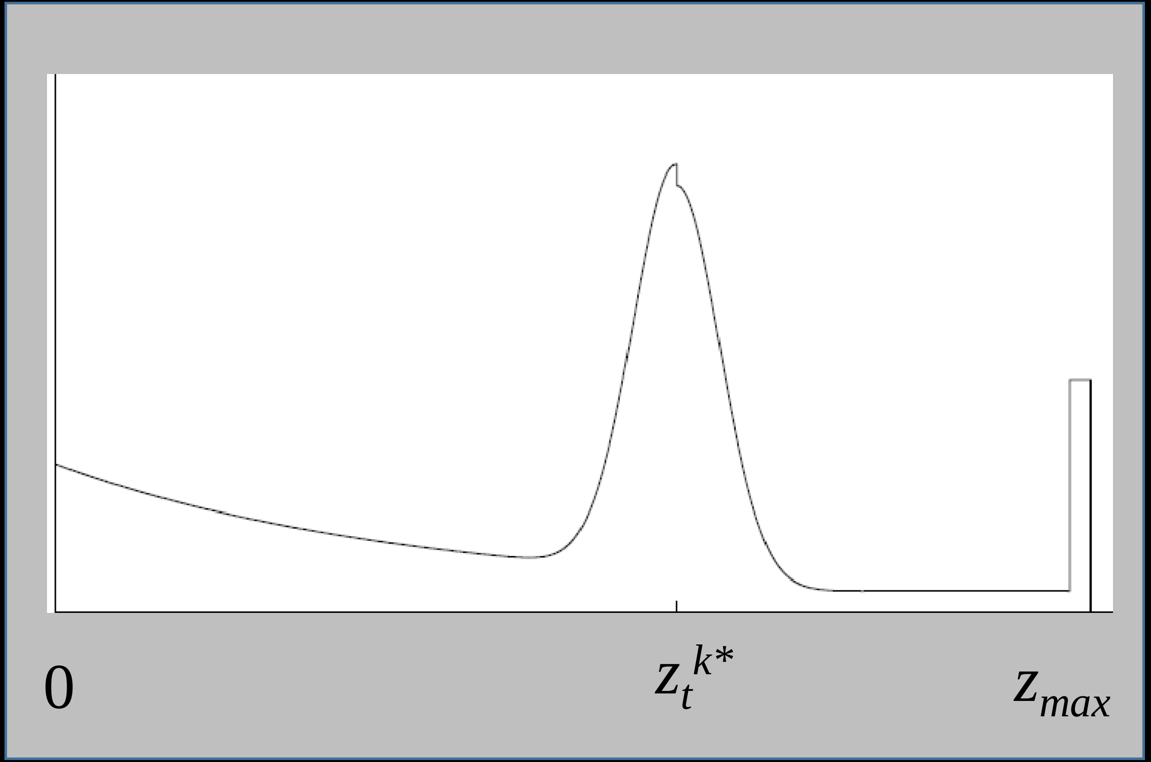


Beam Range Model as a Mixture Density

- The four different distributions are now mixed by a weighted average, defined by the parameters α_{hit} , α_{short} , α_{max} , α_{rand} s.t:

$$\alpha_{hit} + \alpha_{short} + \alpha_{max} + \alpha_{rand} = 1$$

$$p(z_t^k | x_t, m) = \begin{pmatrix} \alpha_{hit} \\ \alpha_{short} \\ \alpha_{max} \\ \alpha_{rand} \end{pmatrix} \cdot \begin{pmatrix} p_{hit}(z_t^k | x_t, m) \\ p_{short}(z_t^k | x_t, m) \\ p_{max}(z_t^k | x_t, m) \\ p_{rant}(z_t^k | x_t, m) \end{pmatrix}$$



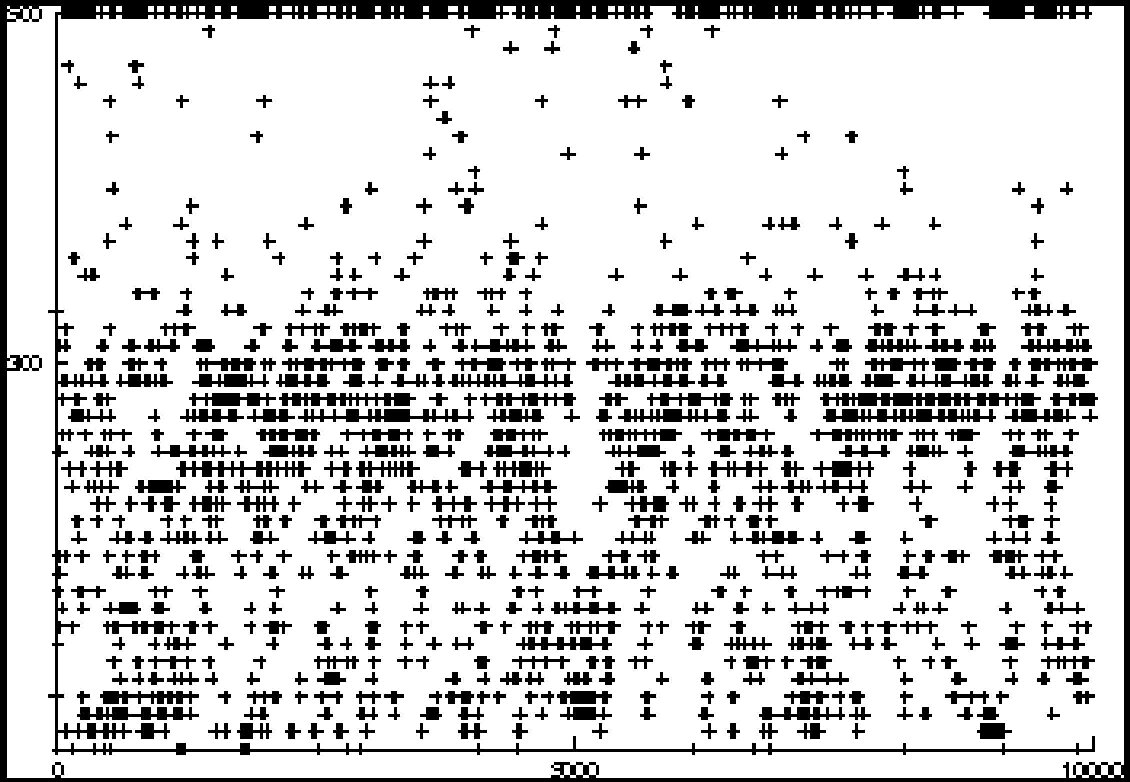
Algorithm for Beam Model

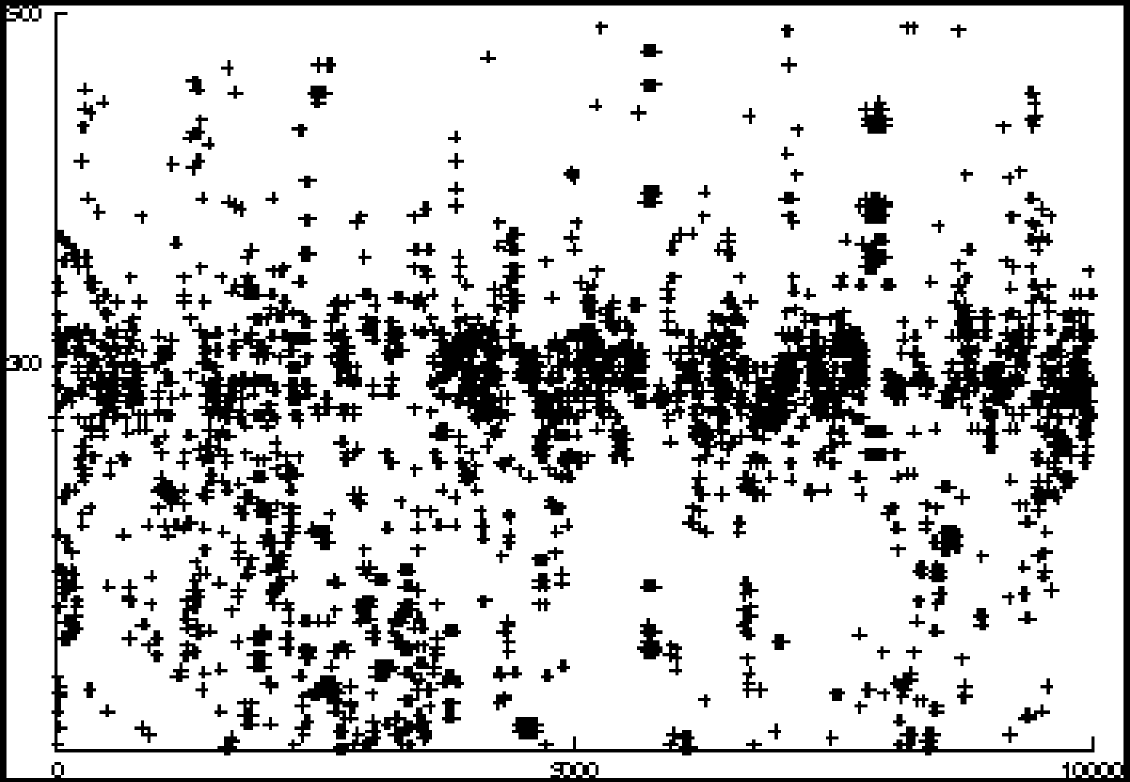
1. **Algorithm beam_range_finder_model**(z_t , x_t , m):
2. $q = 1$
3. for $k = 1$ to K do
4. compute z_t^{k*} for the measurement z_t^k using ray casting
5. $p = \alpha_{hit}.\, p_{hit}(z_t^k | x_t, m) + \alpha_{short}.\, p_{short}(z_t^k | x_t, m)$
6. $+ \alpha_{max}.\, p_{max}(z_t^k | x_t, m) + \alpha_{rand}.\, p_{rand}(z_t^k | x_t, m)$
7. $q = q.p$
8. return q

Parameters of Beam Range Model

- Intrinsic Parameters Θ of the beam range model include α_{hit} , α_{short} , α_{max} , α_{rand} , λ_{short}
- Likelihood of any sensor measurement is a function of Θ
- Estimation Methods:
  - We can guesstimate the resulting density until it agrees with experience
  - Learn parameters using a Maximum Likelihood Estimator that maximizes the likelihood of data for the intrinsic parameter
  - Other methods to estimate parameters: Hill Climbing, Gradient descent, Genetic algorithms, etc

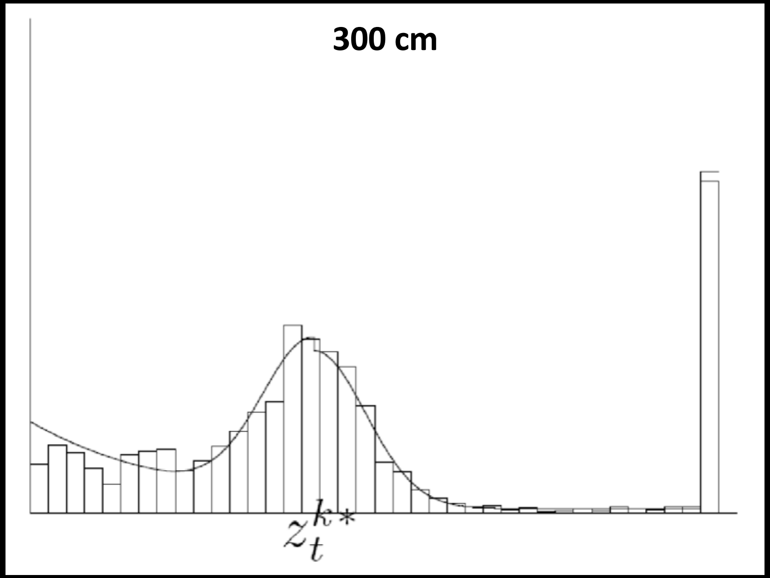
Raw Sensor Data



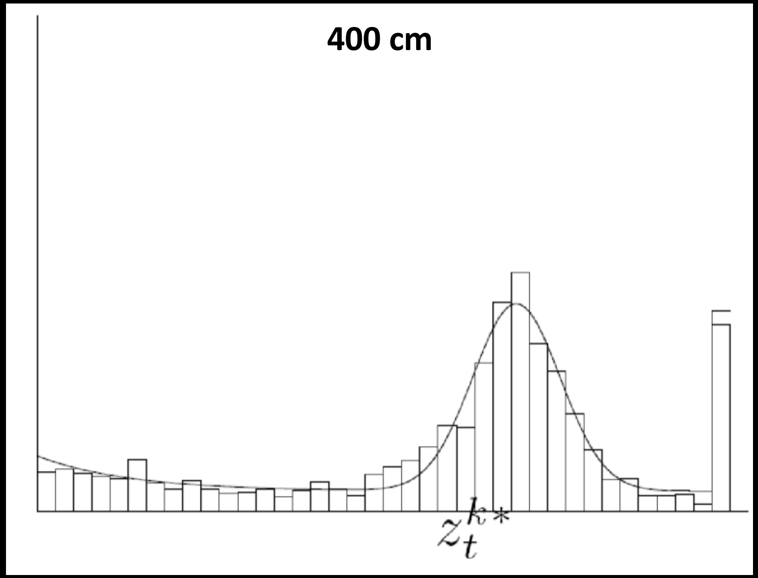


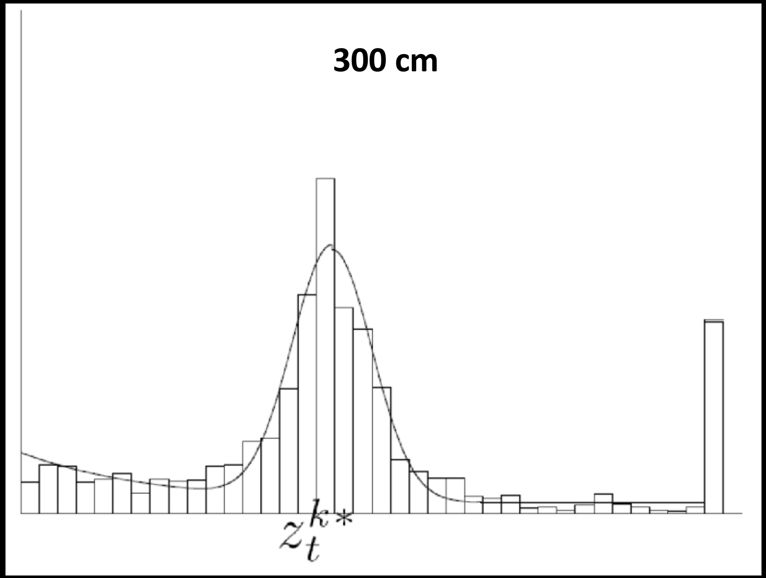
Typical data obtained with (a) a sonar sensor and (b) a laser-range sensor in an office environment for a "true" range of 300 cm and a maximum range of 500 cm

Approximation Results of Beam Model

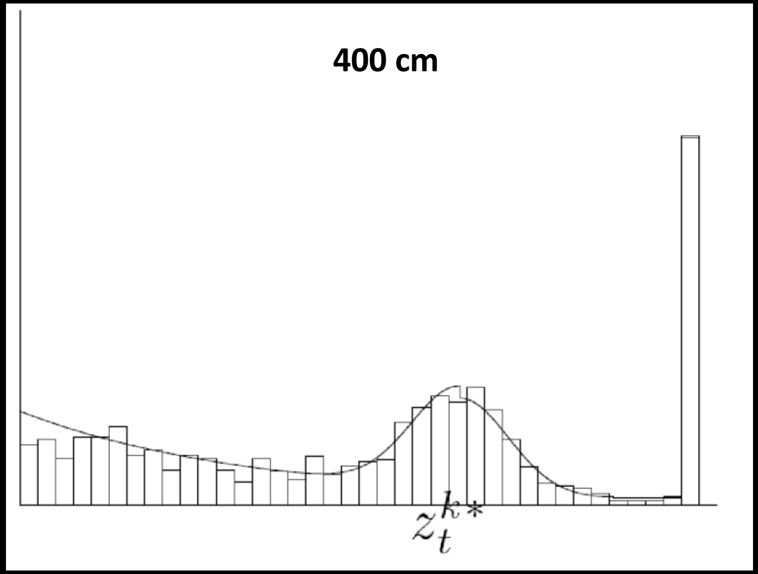


Laser
Data



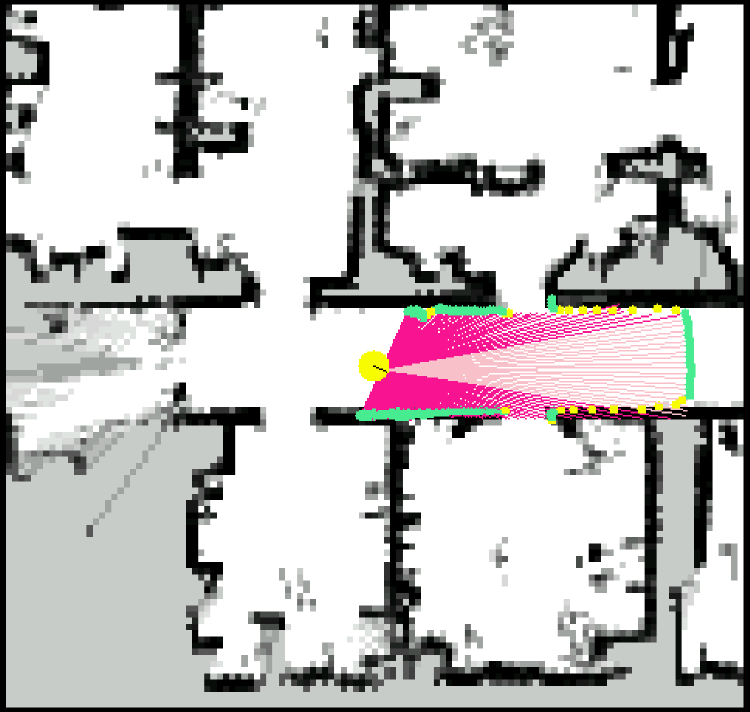


Sonar
Data

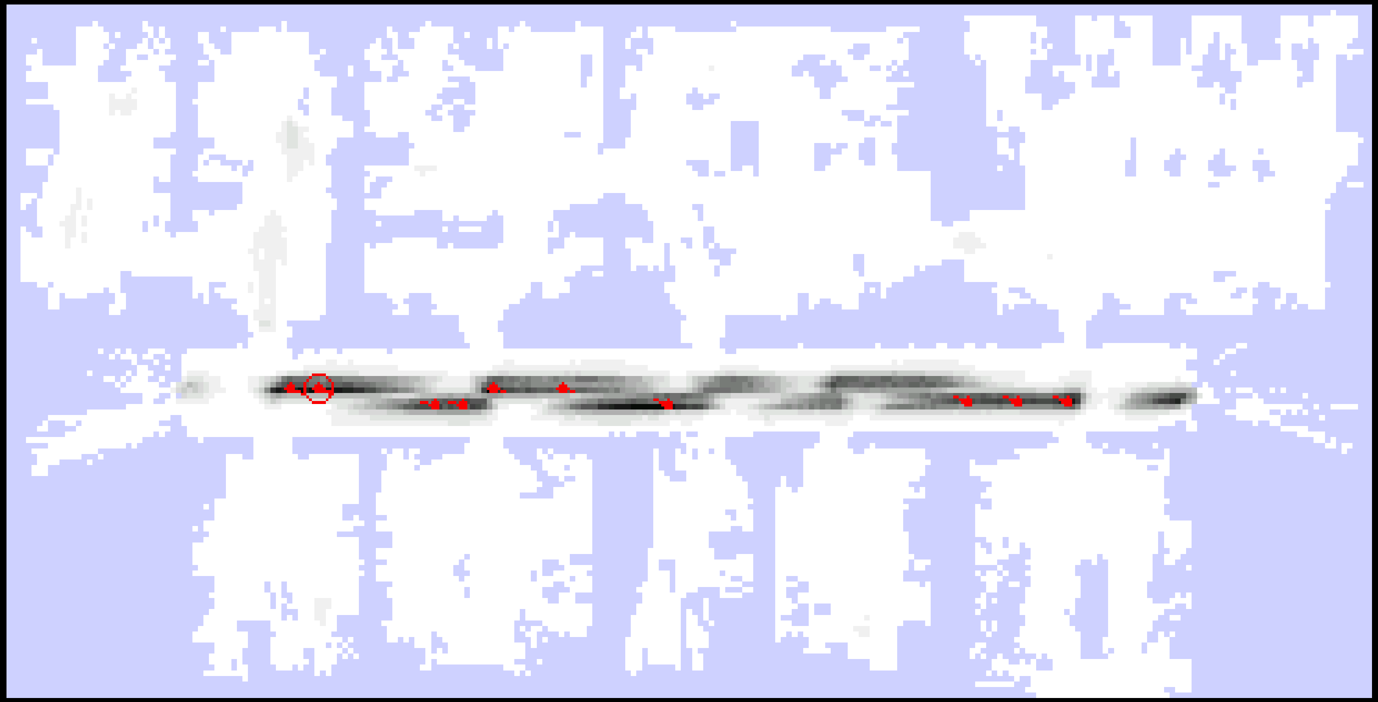


Depicts four examples of data and ML measurement models calculated using a maximum likelihood estimation method.
The left images depict the data from the previous slide.

Learned Probabilistic Sensor Model in Action



(a) Laser scan projected into a previously acquired map m. (Only a portion of the map is depicted)



(b) Likelihood $p(z_t|x_t, m)$ evaluated for all positions x_t and projects into the map (shown in gray). The darker a position, the larger $p(z_t|x_t, m)$

Summary of Beam Model

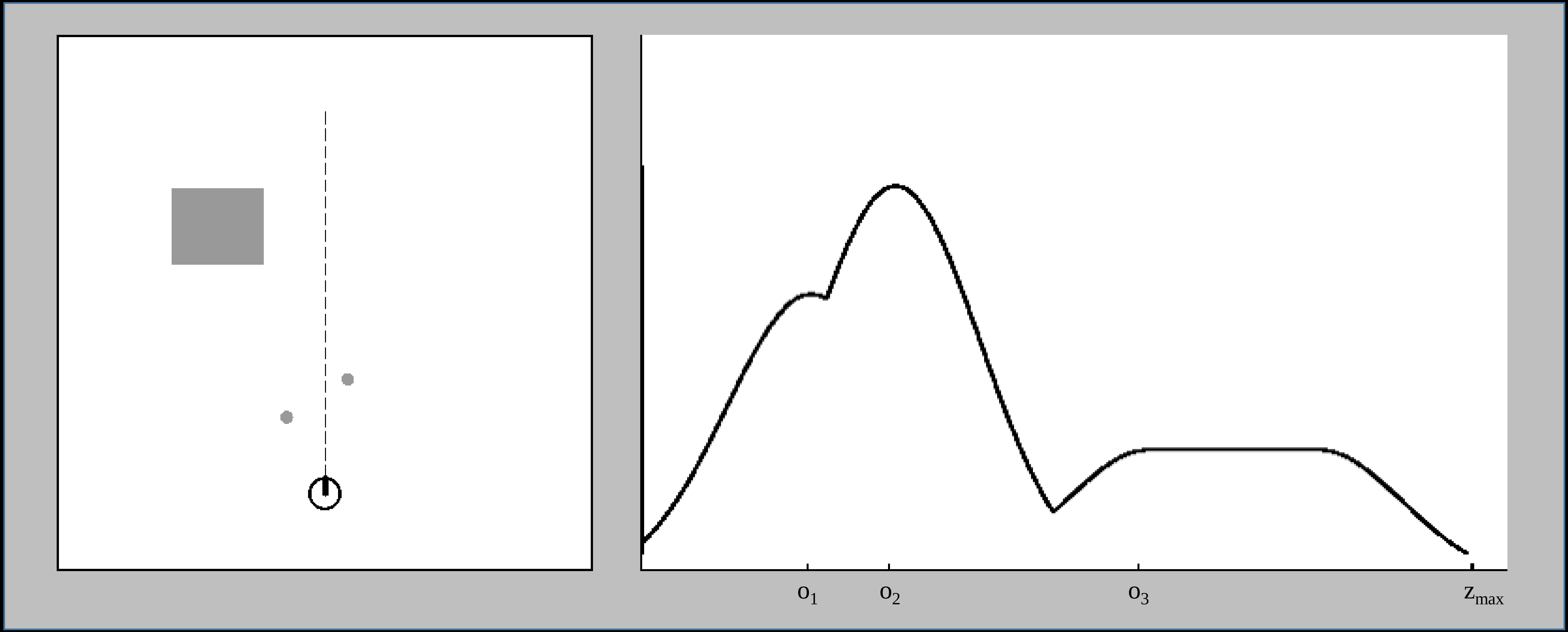
- Overconfident: Assumes independence between beams
- Models physical causes for measurements
- Implementation involves learning parameters based on real data
- Limitations:
  - Different models should be learned for different angles at which the sensor beam hits the obstacle
  - Determine expected distances by ray-tracing is computationally expensive
    - Expected distances can be pre-processed
  - Not smooth for small obstacles, at edges and in cluttered environments

Likelihood Fields

Likelihood Fields of Range Finders

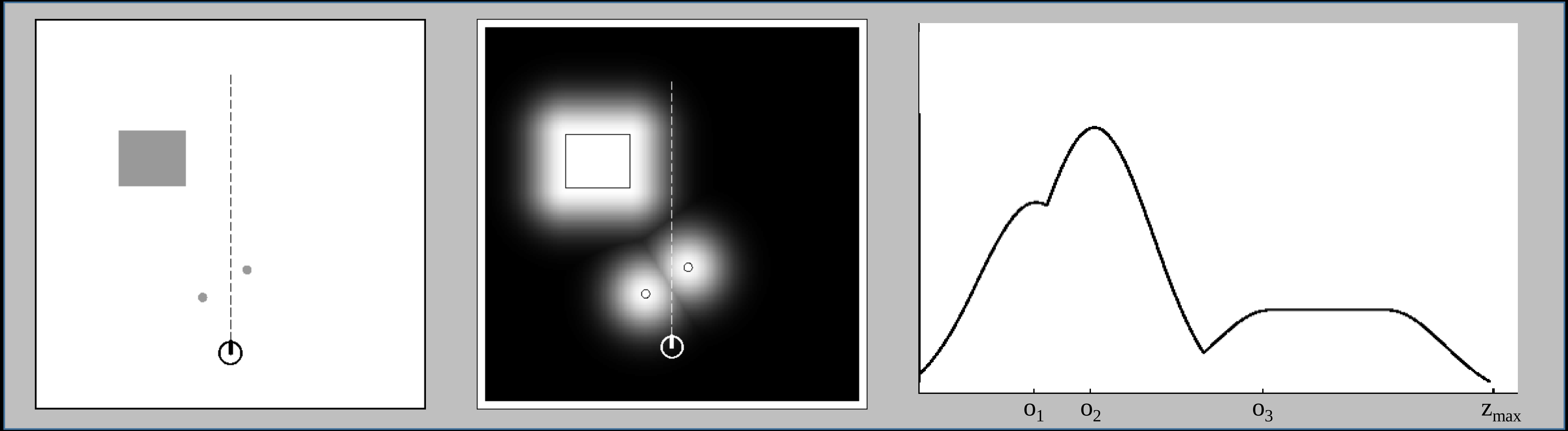
- Lacks a plausible physical explanation
  - No generative model derived from physical interactions to calculate the conditional probability from
- Overcomes some of the limitations of the Beam model
- Instead of following along the beam, just check the end point
- Project sensor scan z_t into the map
  - Requires knowledge of the robot pose in a global coordinate frame
  - Requires knowledge of the sensor beam pose relative to the robot frame

Measurement Noise



Measurement Noise

- Modelled using Gaussians
- In xy space, this involves finding the nearest obstacle in the map
- If *dist* denotes euclidean distance between measurement coordinates and nearest object in the map m, then probability of a sensor measurement is given by the Gaussian $\mathcal{N}(0, dist)$



(a) Example environment with three obstacles (gray), Robot takes a measurement z_k^t (dashed line)
(b) Likelihood filed for this obstacle configuration: darker a location, less likely it is to perceive an obstacle there
(c) Probability $p_{hit}(z_k^t)$ as a function of the measurement z_k^t . The sensor beam passes by three obstacles with respective nearest points o_1, o_2, o_3

Likelihood Fields for Range Finders

- Robot pose $x_t = (x, y, \theta)^T$
- Relative location of the sensor in the robot's frame as $(x_{k,sens}\ ,\ y_{k,sens})$ and the angle of orientation w.r.t. robot's heading direction as $\theta_{k,sens}$
- "End points" of the measurement z_t^k in the global coordinate frame:

$$\begin{pmatrix} x_{z_t^k} \\ y_{z_t^k} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x_{k,sens} \\ y_{k,sens} \end{pmatrix} + z_t^k \begin{pmatrix} \cos(\theta + \theta_{k,sens}) \\ \sin(\theta + \theta_{k,sens}) \end{pmatrix}$$

- Likelihood model rejects measurements $z_t^k = z_{max}$

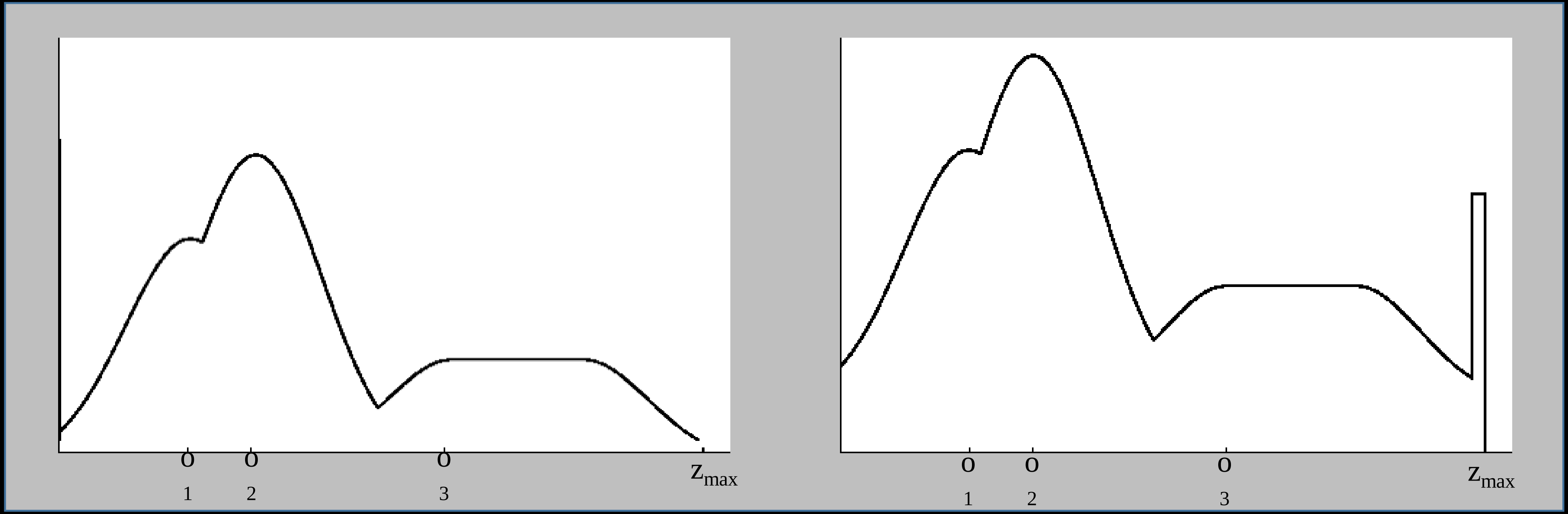
Likelihood Fields of Range Finders

- Independence between different individual measurement values is assumed
- Three types of sources of noise and uncertainty:
    - Measurement Noise
    - Failures
    - Unexplained Random Measurements

Failures and Random Measurements

2. Failures: As before, assume that max range readings have a distinct large likelihood modelled by a point-mass distribution

3. Unexplained Random Measurements: A uniform distribution is used to model random noise in perception



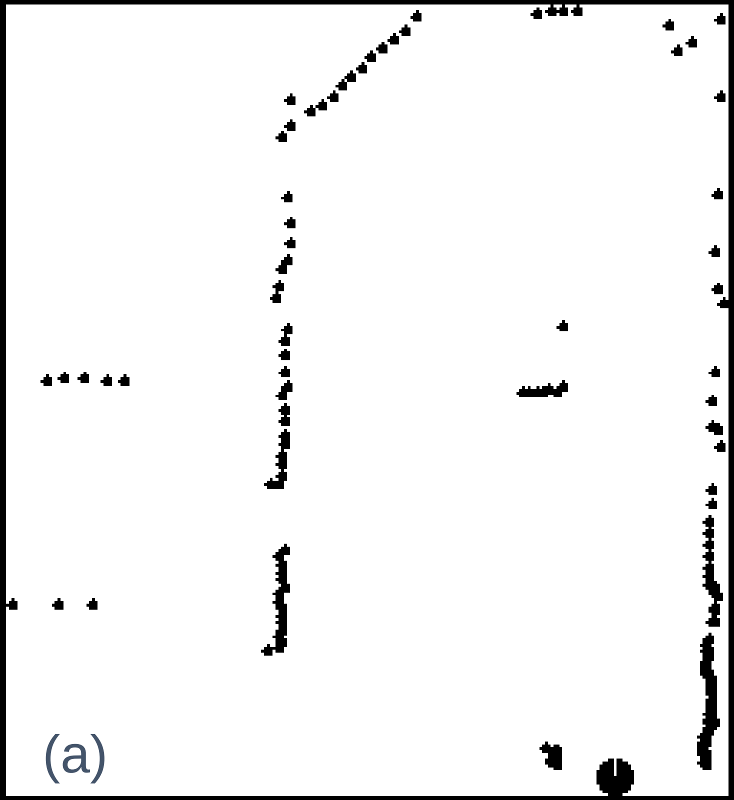
(a) Probability $p_{hit}(z_k^t)$ as a function of the measurement z_k^t .The sensor beam passes by three obstacles with respective nearest points o_1, o_2, o_3

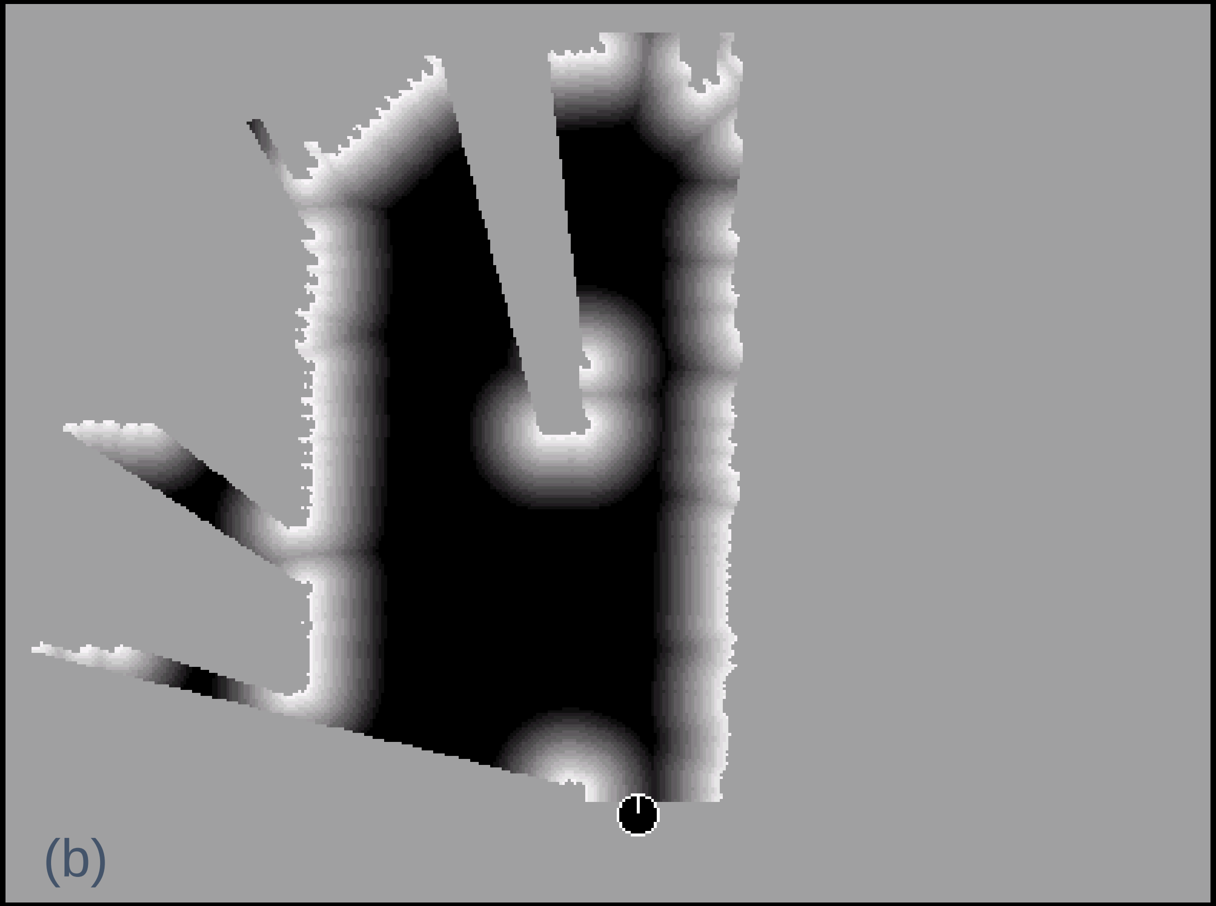
(b) Sensor probability obtained for the situation depicted in the previous slide by incorporating for failures and random measurements

Algorithm for Beam Model

1. **Algorithm likelihood_field_range_finder_model**(z_t, x_t, m):
2. $q = 1$
3. for $k = 1$ $to\ K\ do$
4. $x_{z_t^k} = x + x_{k,sens}\cos(\theta) - y_{k,sens}\sin(\theta) + z_t^k\cos(\theta + \theta_{k,sens})$
5. $y_{z_t^k} = y + y_{k,sens}\cos(\theta) + x_{k,sens}\sin(\theta) + z_t^k\sin(\theta + \theta_{k,sens})$
6. $dist = \min_{x',y'}\left\{\sqrt{(x_{z_t^k} - x')^2 + (y_{z_t^k} - y')^2} \mid \langle x', y'\rangle \text{ occupied in } m\right\}$
7. $q = q.(z_{hit}.\mathcal{N}(dist; 0, \sigma_{hit}) + \frac{z_{rand}}{z_{max}})$
8. return q

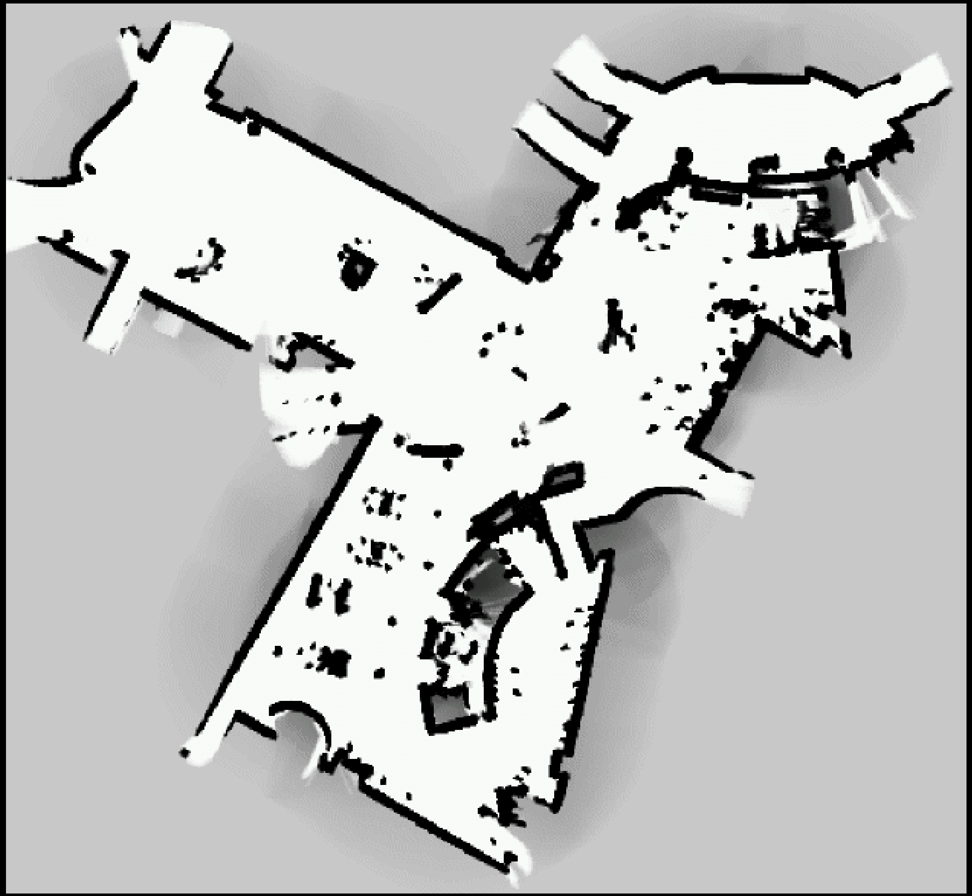
Likelihood Field from Sensor Data



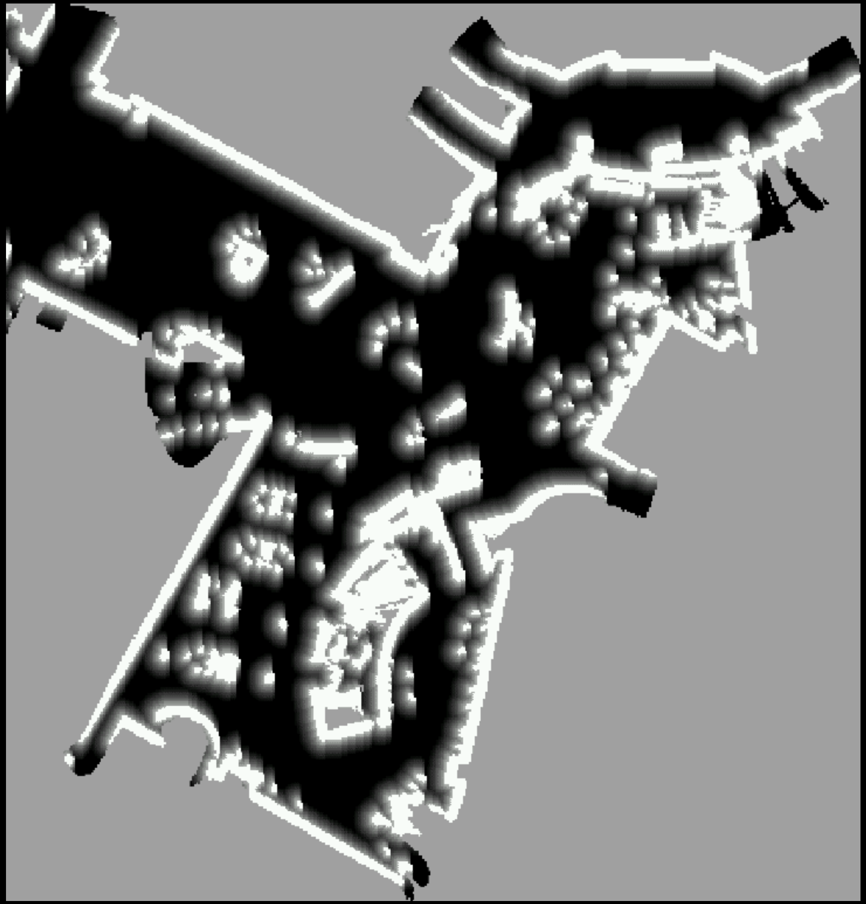


(a) Sensor data consisting of 180 dots visualized from a bird's eye perspective
(b) Likelihood function generated from this sensor scan (darker a region, smaller the likelihood for sensing an object there.

San Jose Tech Museum



Occupancy grid map



Likelihood field

Summary of Likelihood Fields

- Probability is a mixture of:
  - a Gaussian distribution with mean at distance to closest obstacle
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements.
- Again, independence between different components is assumed.

Summary of Likelihood Fields

- Advantages:
  - Highly efficient, computation in 2D instead of 3D
  - Smooth w.r.t. to small changes in robot position (due to euclidean distance)
- Limitations:
  - Does not model people and other dynamics that might cause short readings
  - Ignores physical properties of beams: Model can "see through walls" as likelihood fields are incapable of determining whether a path to a point is intercepted by an obstacle in the map

Feature Based Models

Feature Based Models

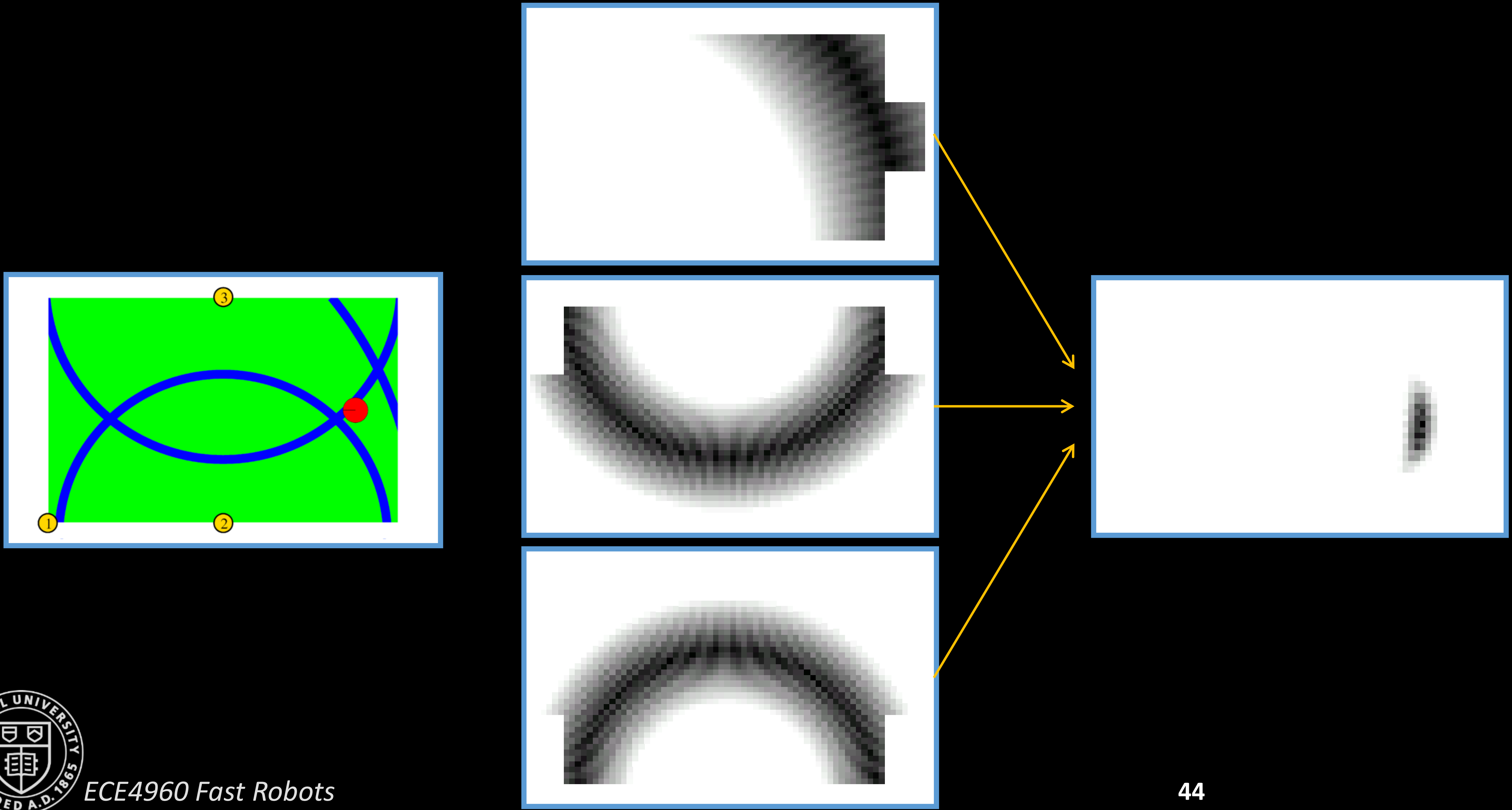
- Sensor models discussed so far are based on raw sensor measurements
- Alternative approach is to extract features (usually smaller in number) from dense raw measurements
- Inference in the (sparser) feature space can be more efficient
- For range sensors, features such as lines, corners, etc may be extracted
- Myriad of feature extraction methods from camera images (edges, corners, distinct patterns, etc)
- In robotics, features correspond to distinct physical objects in the real world and are often referred to as *landmarks*

Landmarks

- Sensors (generally) measure the range (distance) and bearing (angle) of the landmark w.r.t to the robot's frame
  - Active beacons (e.g., radio, GPS)
  - Passive (e.g., visual, retro-reflective)
- Sensors may provide:
  - Range
  - Bearing
  - Range and Bearing

Trilateration using Range Measurements

Range and Bearing Distributions



Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness
- In many cases, good models can be found by the following approach:
    1. Determine parametric model of noise free measurement
    2. Analyze sources of noise
    3. Add adequate noise to parameters (eventually mix in densities for noise)
    4. Learn (and verify) parameters by fitting model to data
    5. Likelihood of measurement is given by “probabilistically comparing” the actual with the expected measurement
- This holds for motion models as well
- It is extremely important to be aware of the underlying assumptions!

Reference

1. Thrun, Sebastian, Wolfram Burgard, and Dieter Fox. Probabilistic robotics. MIT press, 2005.
2. http://ais.informatik.uni-freiburg.de/teaching/ss10/robotics/slides/07-sensor-models.pdf
3. http://www.cs.cmu.edu/~16831-f14/notes/F12/16831_lecture03_mtaylormshomin.pdf
4. Gaussian Distribution: https://www.asc.ohio-state.edu/gan.1/teaching/spring04/Chapter3.pdf
5. Gaussian Distribution: http://www2.stat.duke.edu/~rcs46/modern_bayes17/lecturesModernBayes17/lecture-3/03-normal-distribution.pdf
6. Visual Demos:
   1. Prof. Fred Martin: https://www.youtube.com/watch?v=u293629ZwIo
   2. Prof. Myriam Hunink: https://vimeo.com/236607953