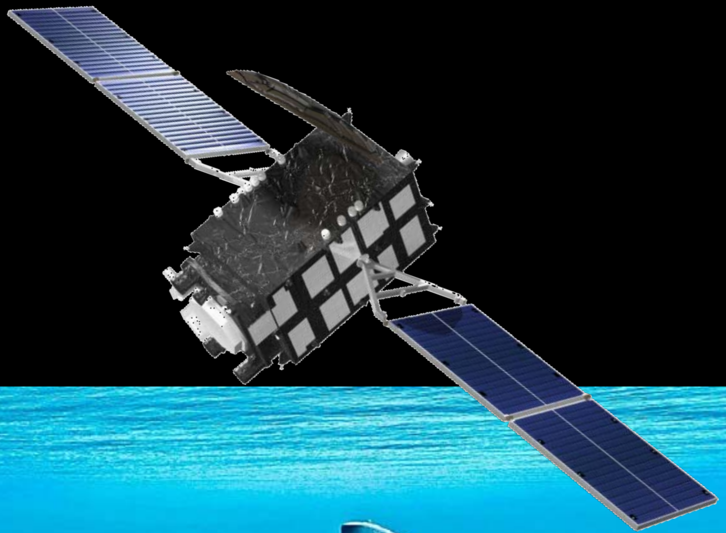
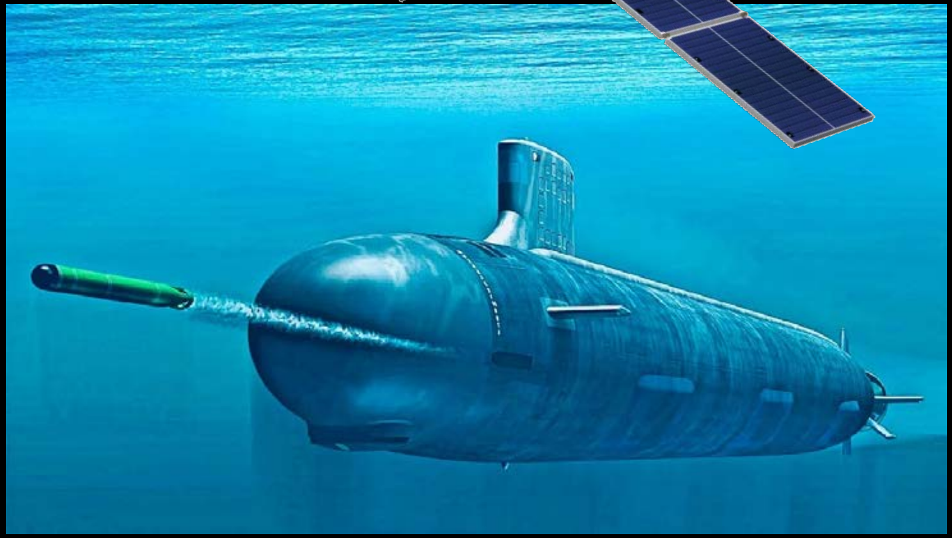
Prof. Kirstin Hagelskjær Petersen
kirstin@cornell.edu

# Fast Robots

# IMU

- Inertial Measurement Unit
  - Data related to orientation, velocity, and gravity



# IMU

- Inertial Measurement Unit
- *Accelerometer*
  - Linear acceleration, $a = \dot{v}$ [m/s²]
- *Gyroscope*
  - Angular velocity, $\dot{\omega} = \frac{\Delta\theta}{\Delta t}$ [deg/sec]
- *Magnetometer*
  - Magnetic field strength, [uT] or [Gauss], (1 Gauss = 100uT)

→ Track orientation (position)

→ Track orientation

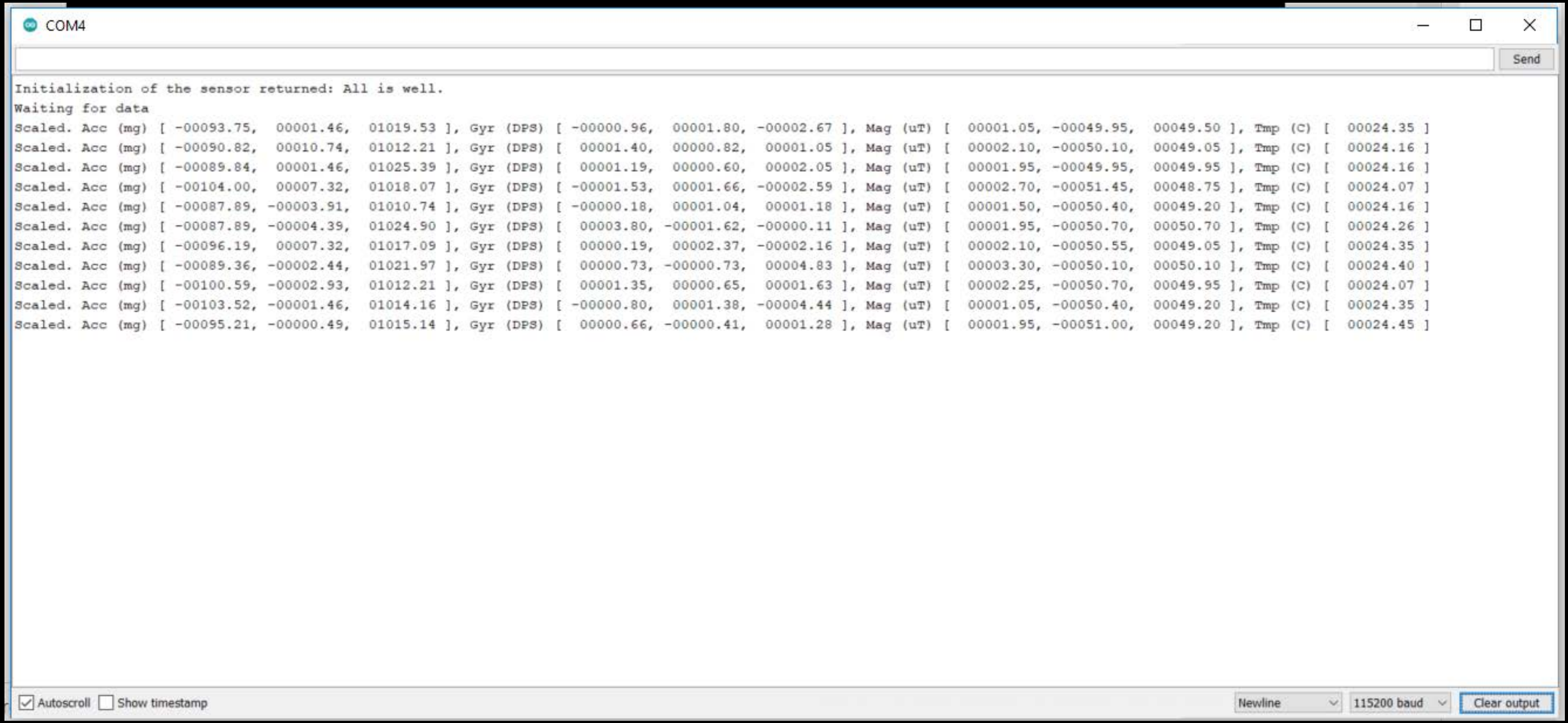
} Dead reckoning

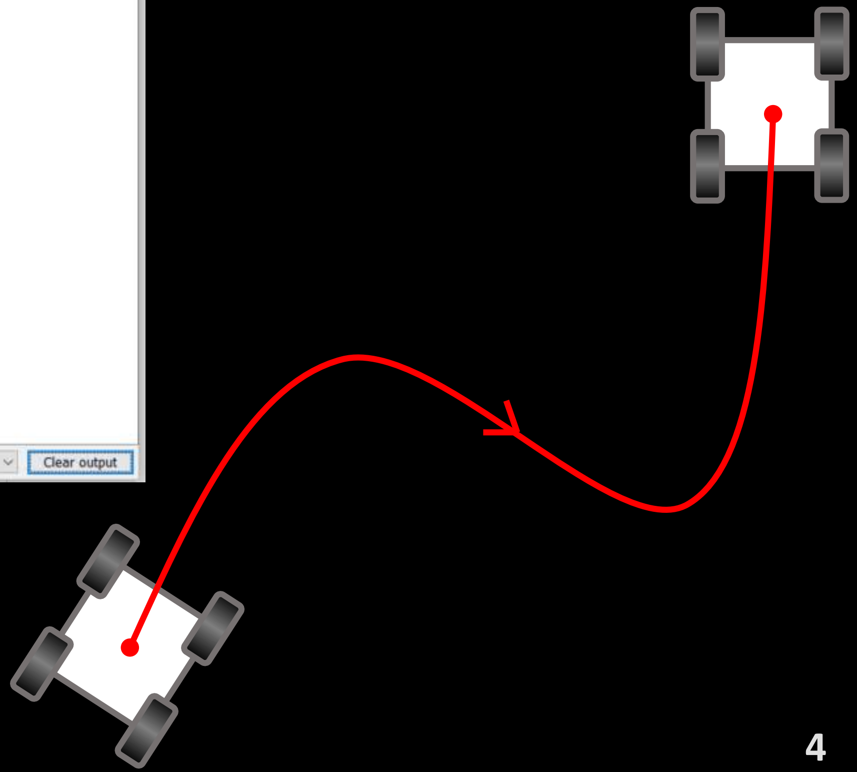
→ Get absolute orientation

- *NB: Gravity, magnetic fields, accelerations affect these sensors in many ways!*

# IMU - Demo

- ..\SparkFun_ICM-20948_ArduinoLibrary-master\examples\Arduino\Example1_Basics



Prof. Kirstin Hagelskjær Petersen
kirstin@cornell.edu

# ACCELEROMETER

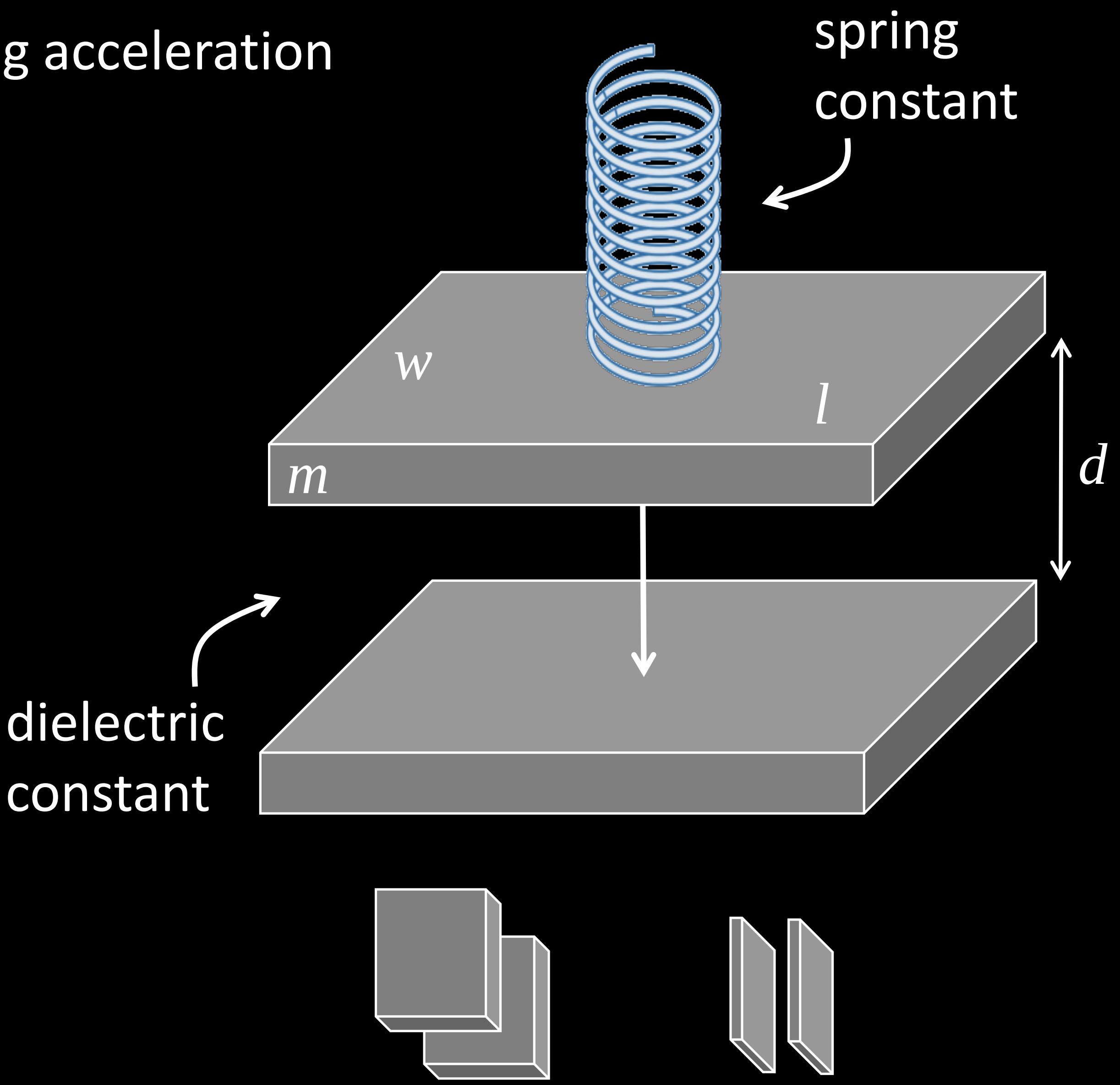
# Accelerometer

- Measuring acceleration

$$C = \frac{\in A}{d}$$

$$F = k\Delta d$$

$$F = ma$$



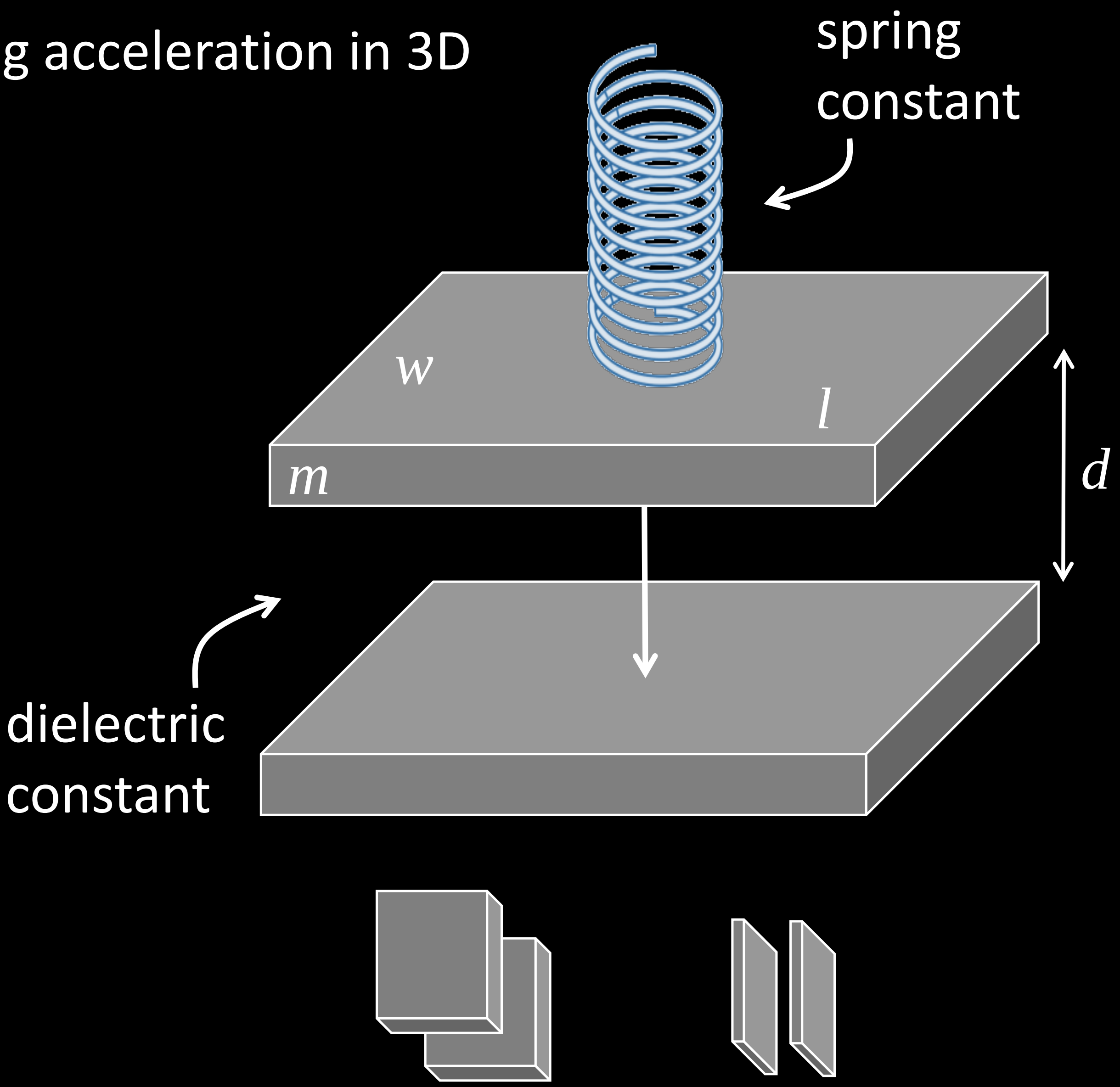
# Accelerometer

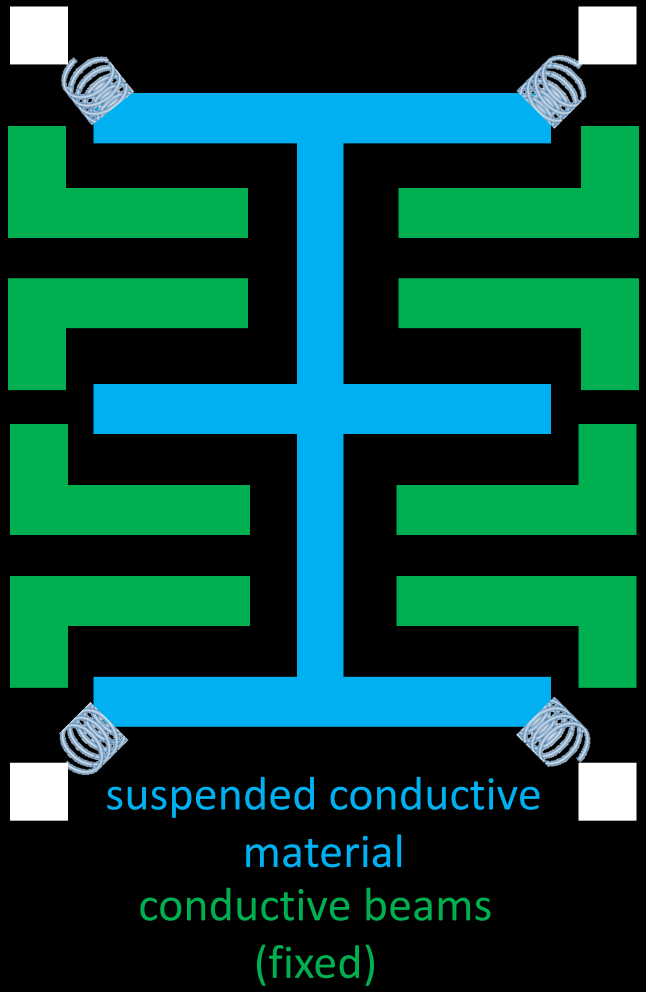
- Measuring acceleration in 3D

$$C = \frac{\in A}{d}$$

$$F = k\Delta d$$

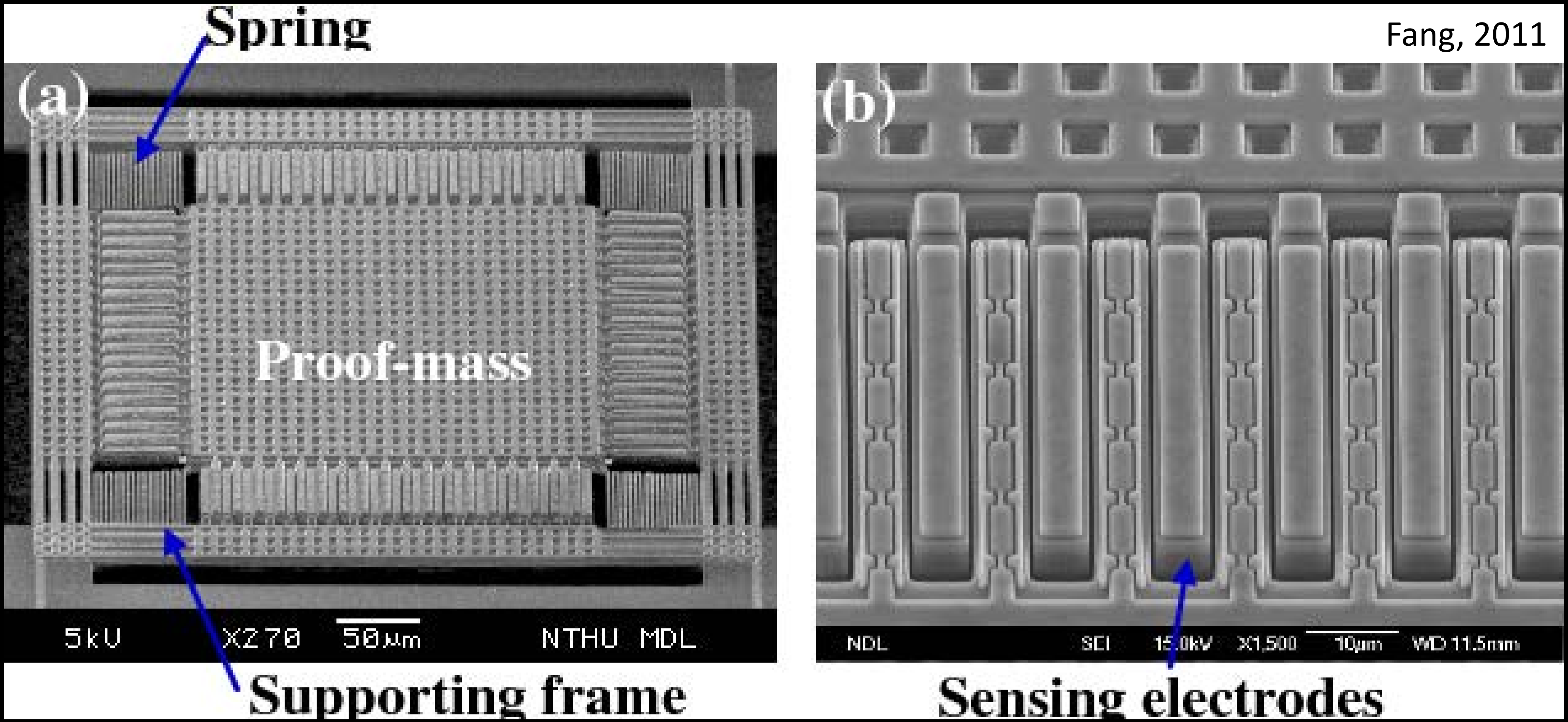
$$F = ma$$



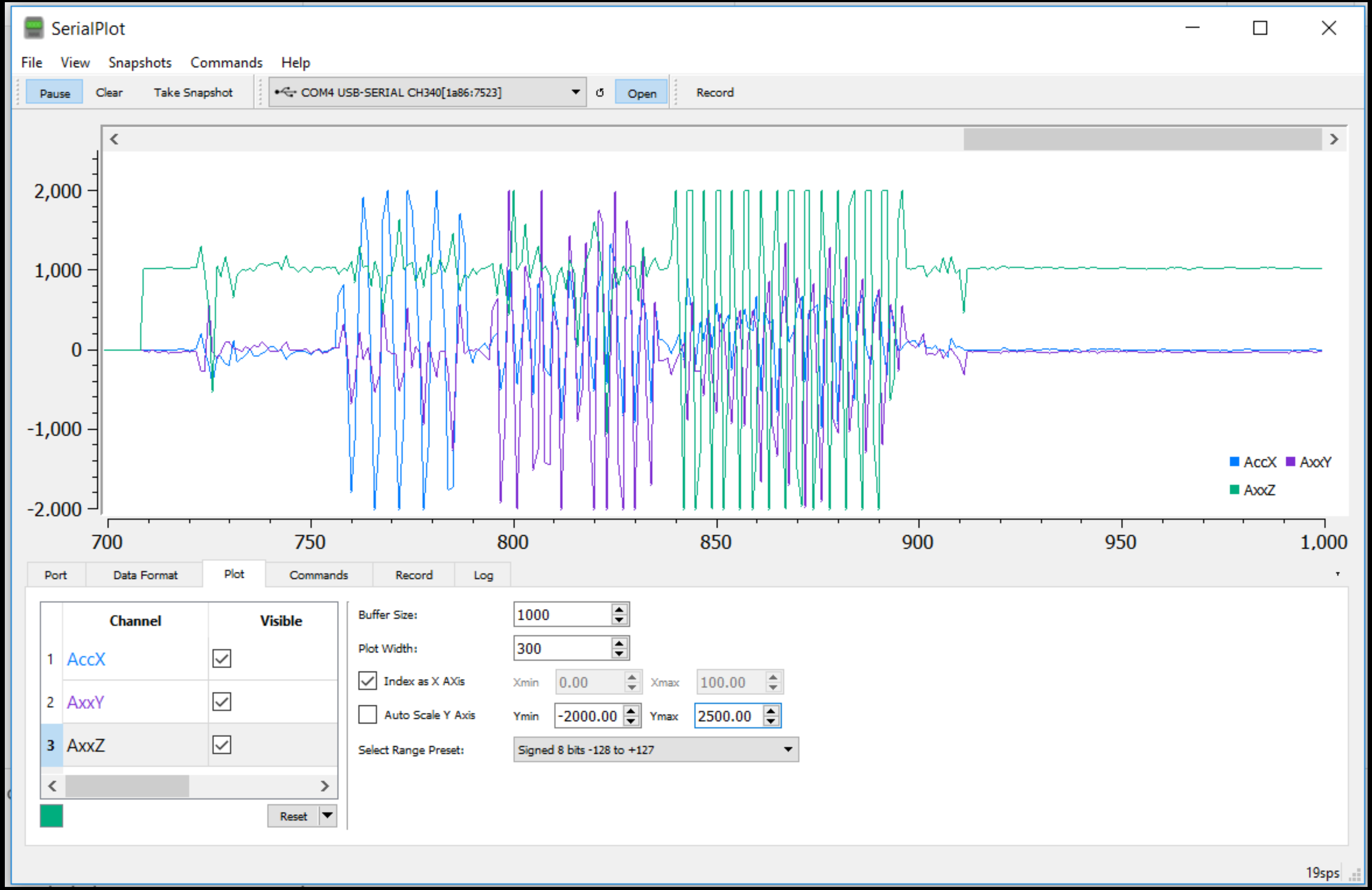
# Accelerometer

- Measuring acceleration in 3D
- Micro-Electro-Mechanical Systems



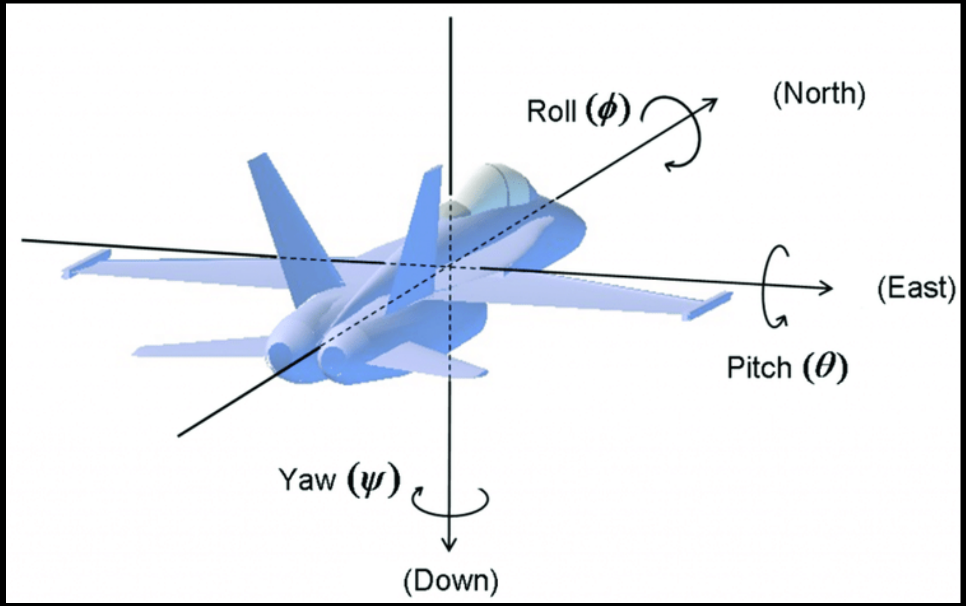
# Accelerometer

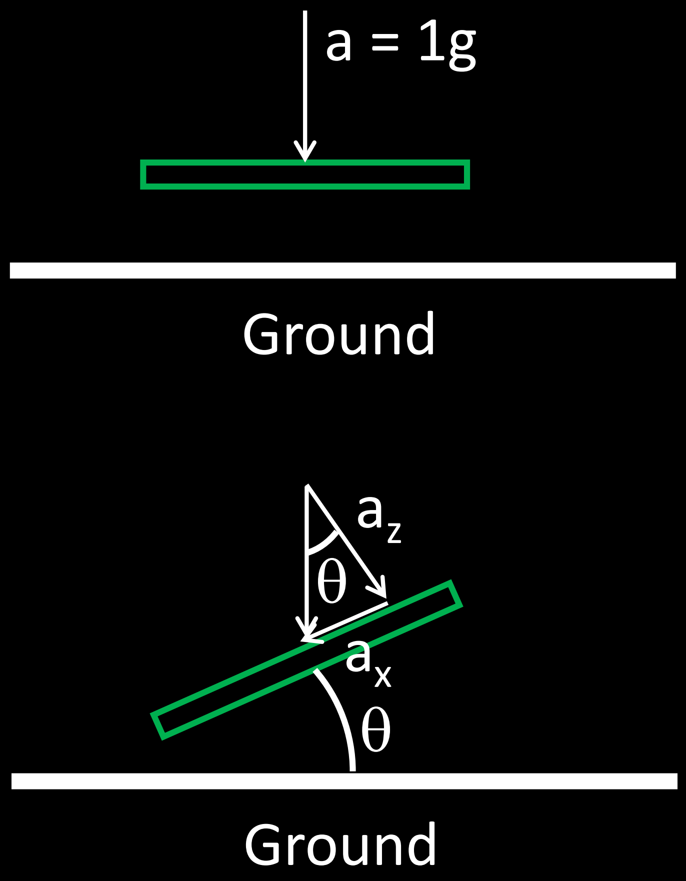
- Measuring acceleration in 3D
- Use a program like SerialPlot to visualize your data



# Accelerometer

- How to use the accelerometer to determine roll, tilt, and ~~yaw~~?



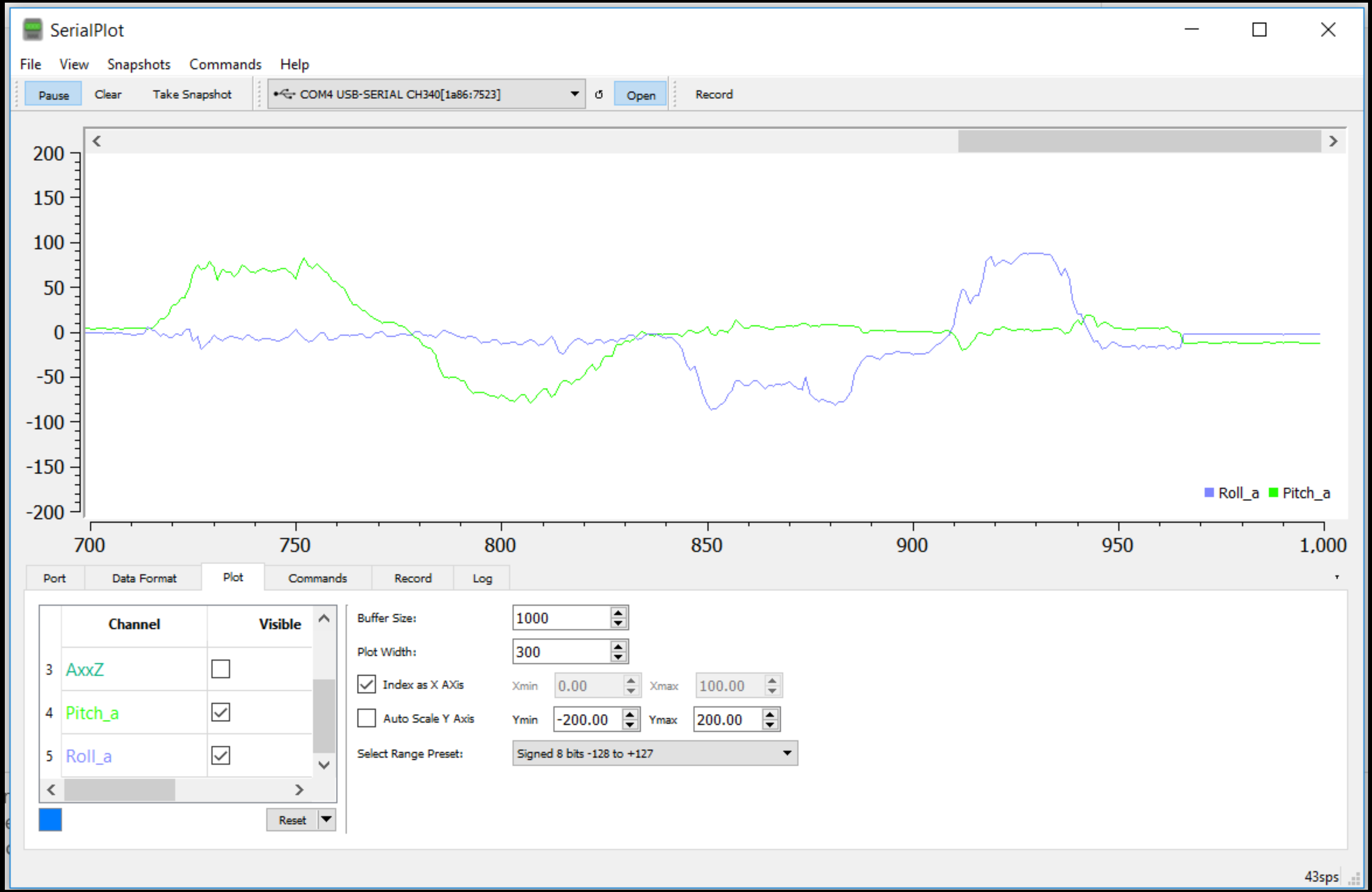




- $a_x = 1g \sin \theta$
- $a_z = 1g \cos \theta$
- $a_x / a_z = \tan \theta$
- $\theta = \text{atan}(a_x/a_z)$
  - Remember, use atan2!
- $\phi = \text{atan}(a_y/a_z)$

**How do you measure yaw with the accelerometer?**

# Accelerometer

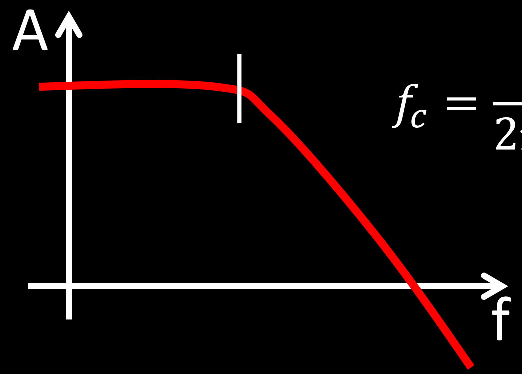
- Determining tilt and roll
- $\theta = \text{atan}(a_x/a_z)$
- $\phi = \text{atan}(a_y/a_z)$



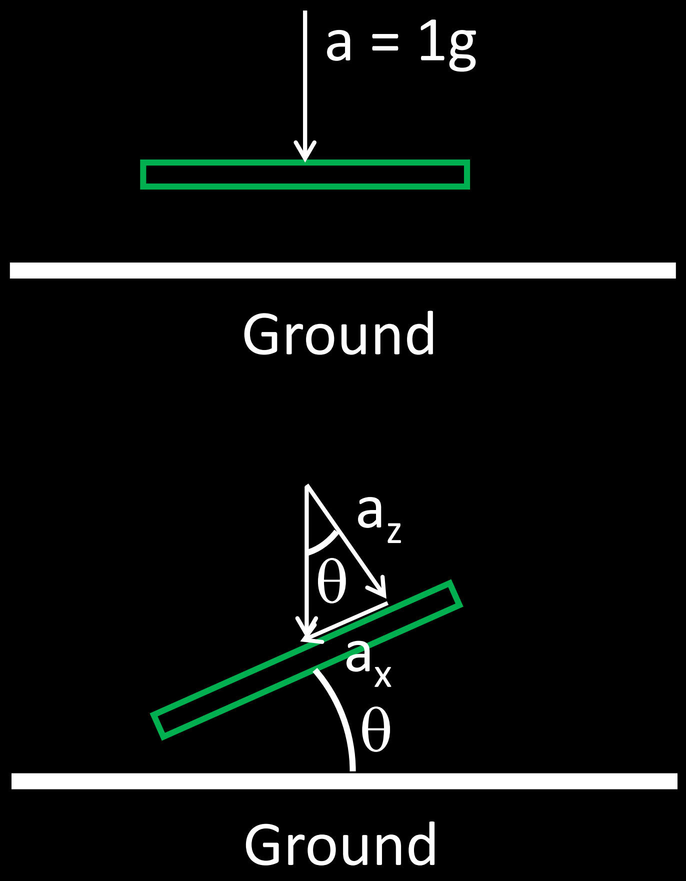
# Accelerometer

- Determining tilt and roll
- Good (very accurate on average) vs Bad (noisy)
- Low pass complimentary filter

  - $\theta_{LPF}[n] = \alpha * \theta + (1 - \alpha) * \theta_{LPF}[n-1]$
  - $\theta_{LPF}[n-1] = \theta_{LPF}[n]$
  - $\alpha = \frac{T}{T+RC}$

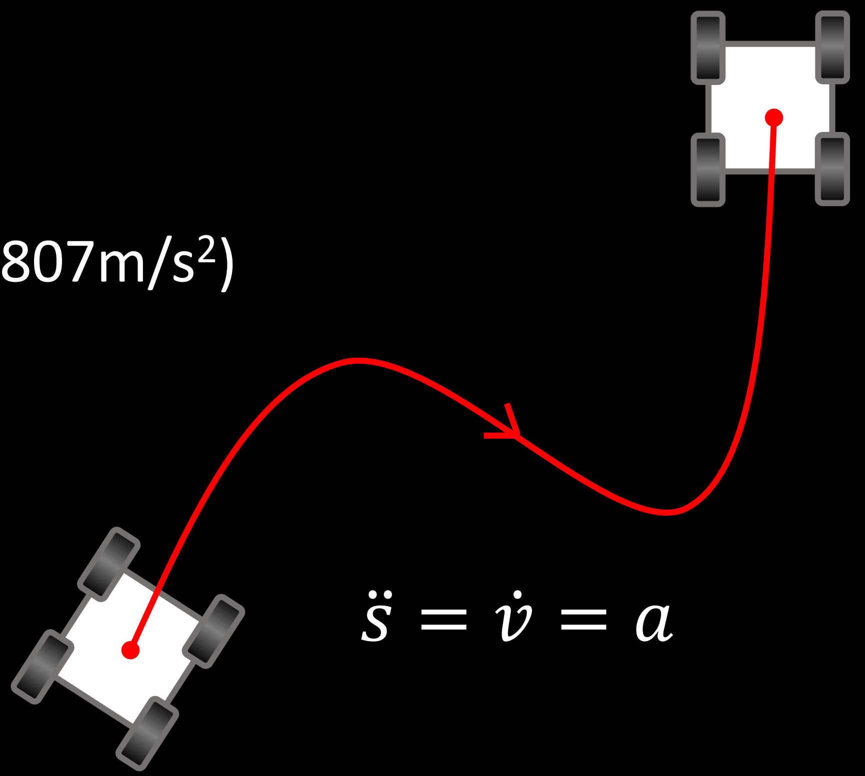



$$f_c = \frac{1}{2\pi RC}$$



# Accelerometer

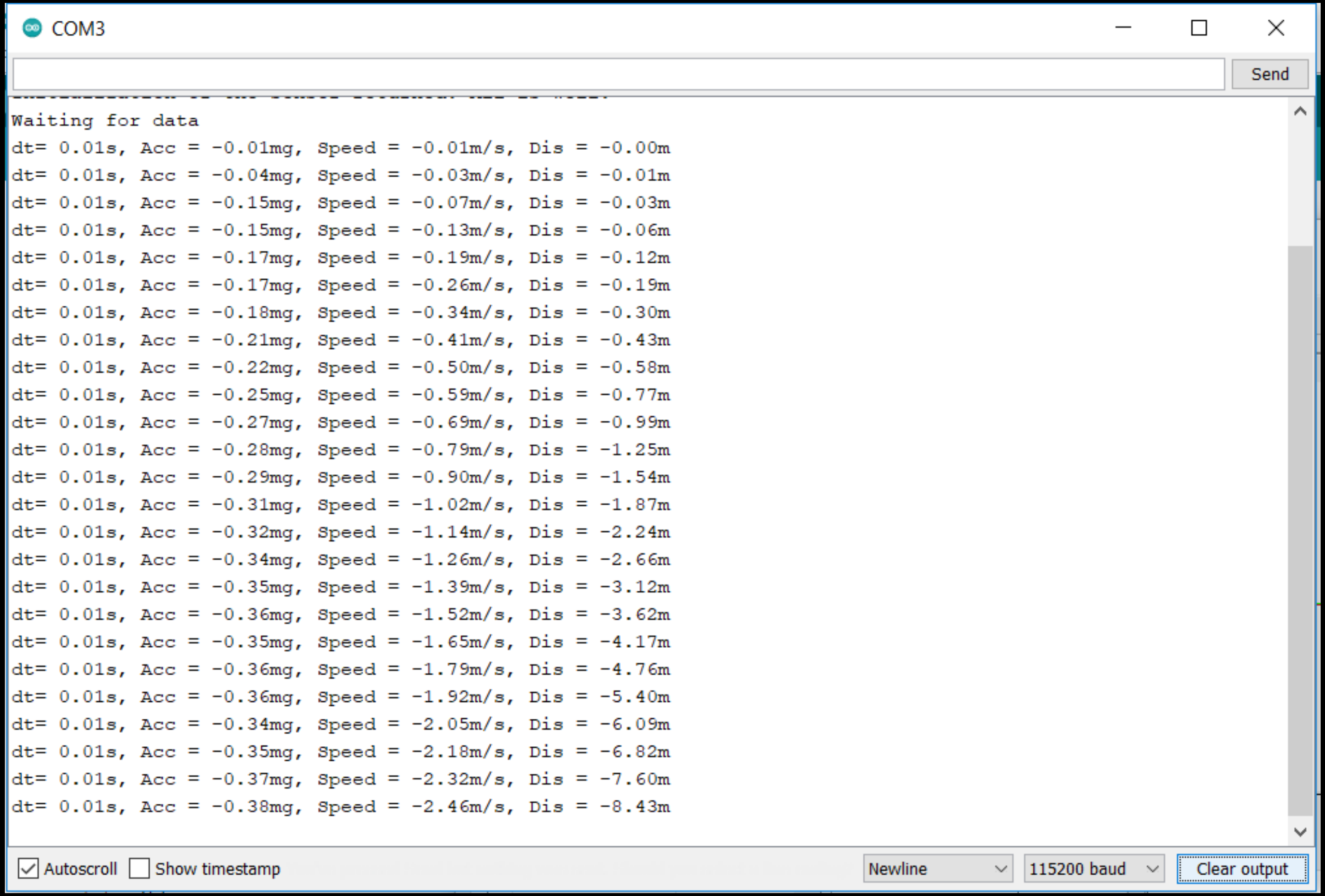
- Determining tilt and roll √
- How to use the accelerometer to do dead reckoning?
  - $v = \int a$
  - s= $\int \int a$
  - `v[k+1]=v[k]+a[k]*dt`
  - `s[k+1]=s[k]+v[k]*dt`
- *Remember:* The accelerometer output is in mg (1g = 9.807m/s$^2$)

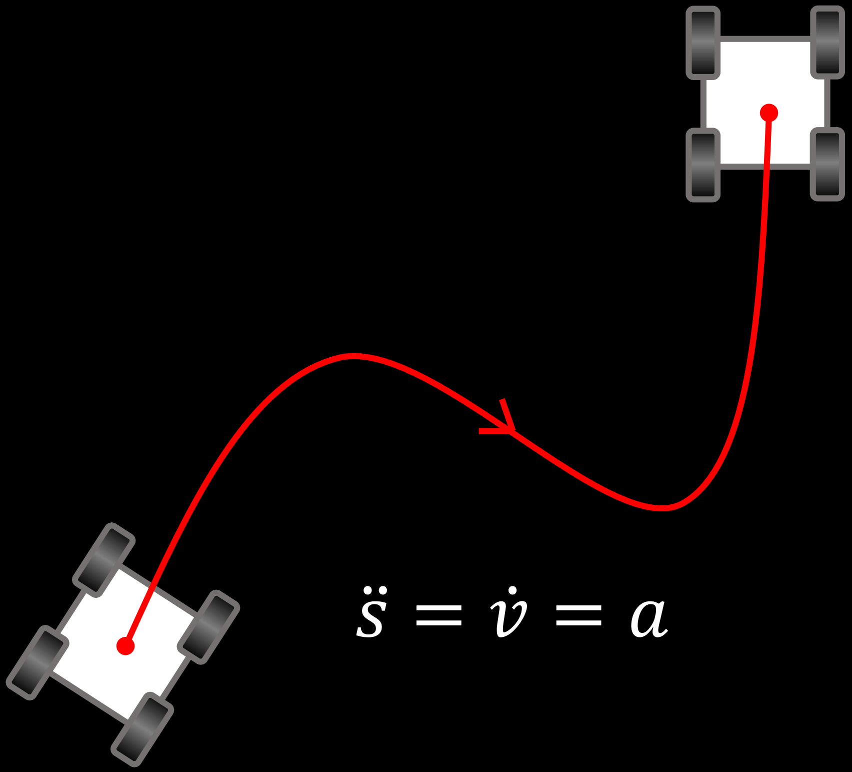


$$\ddot{s} = \dot{v} = a$$

# Accelerometer

- Determining tilt and roll √
- How to use the accelerometer to do dead reckoning?
    - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration

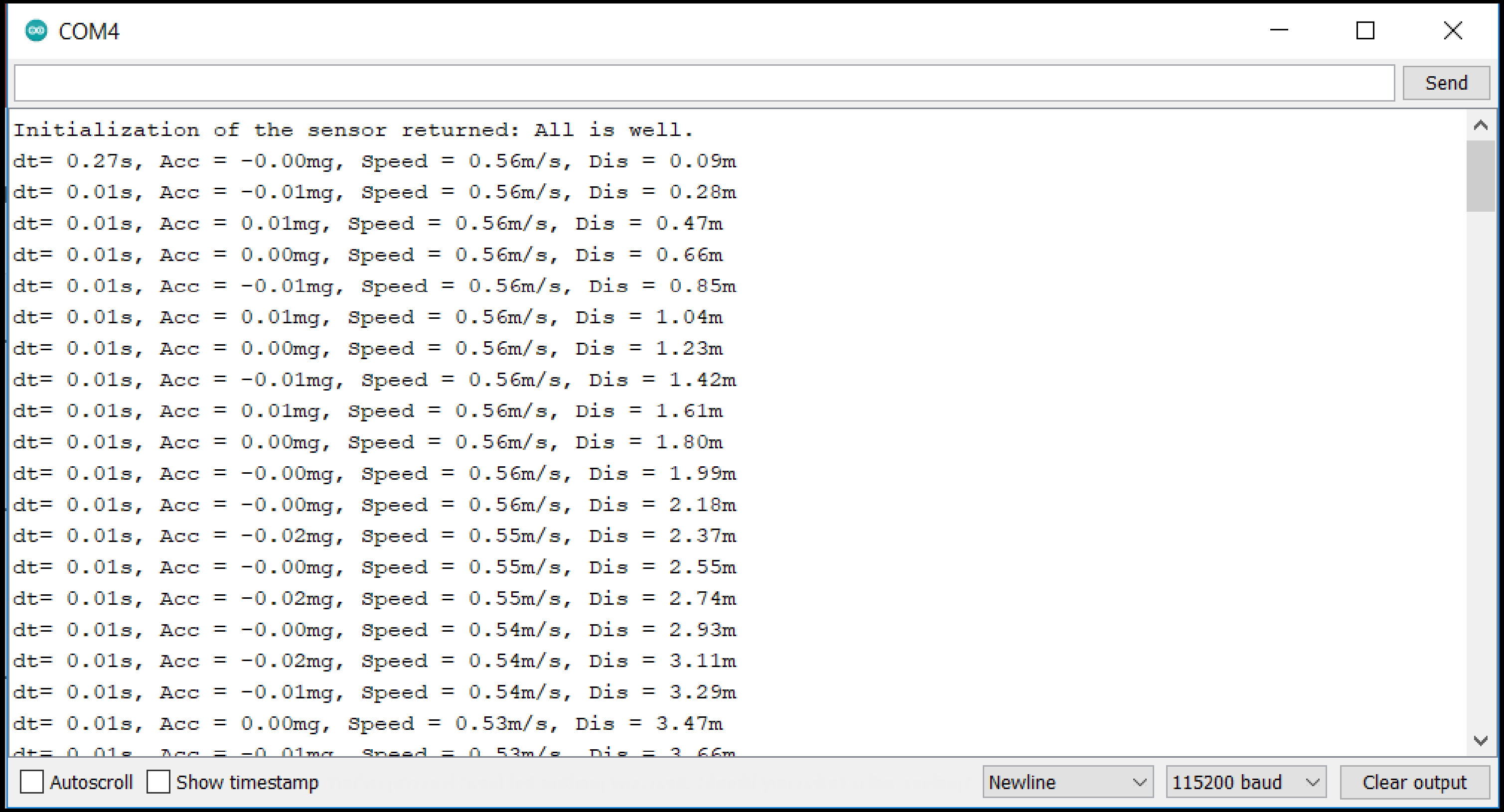


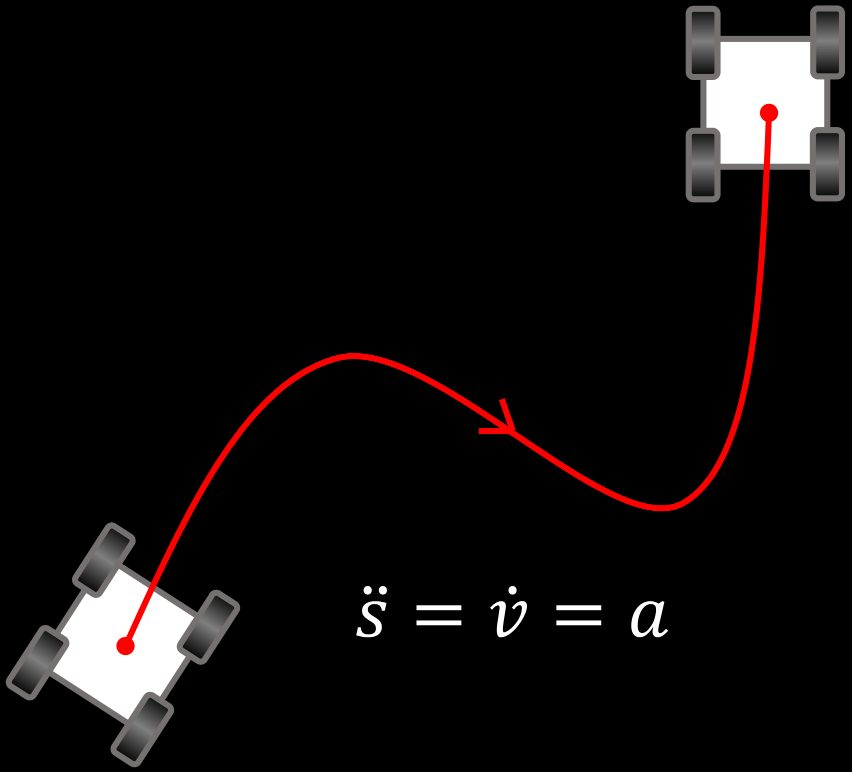



$$\ddot{s} = \dot{v} = a$$

# Accelerometer

- Determining tilt and roll √
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1*: Calibrate the offset

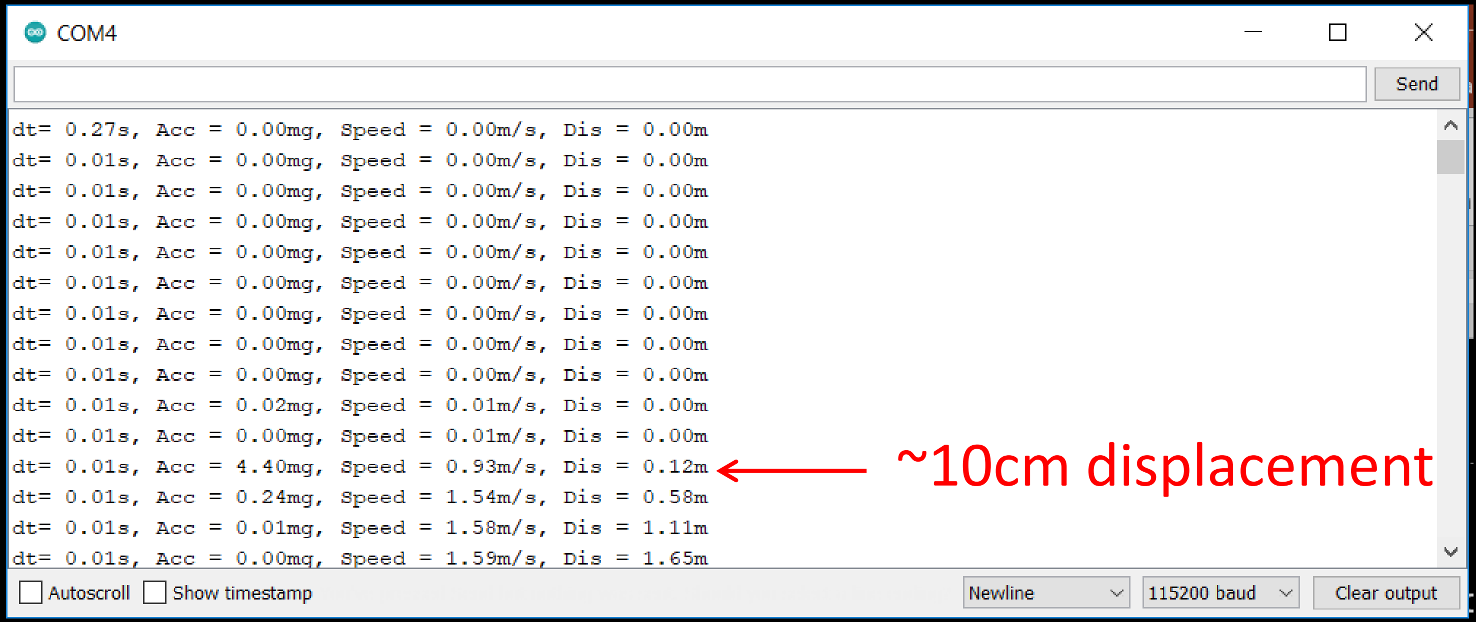


$$\ddot{s} = \dot{v} = a$$

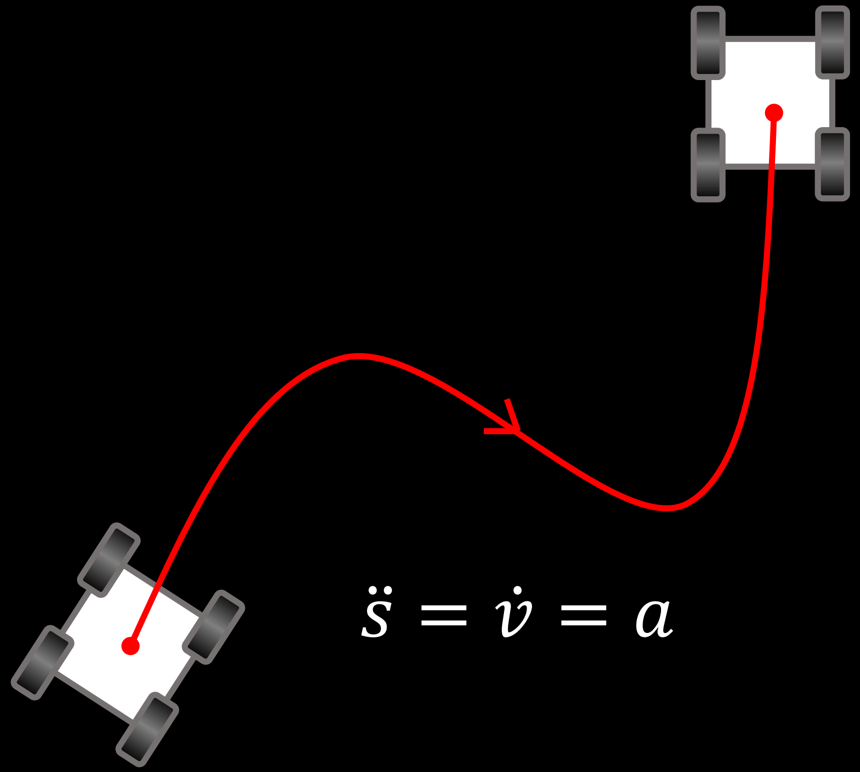
# Accelerometer

- Determining tilt and roll √
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1*: Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off




```
dt= 0.27s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.02mg, Speed = 0.01m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.01m/s, Dis = 0.00m
dt= 0.01s, Acc = 4.40mg, Speed = 0.93m/s, Dis = 0.12m
dt= 0.01s, Acc = 0.24mg, Speed = 1.54m/s, Dis = 0.58m
dt= 0.01s, Acc = 0.01mg, Speed = 1.58m/s, Dis = 1.11m
dt= 0.01s, Acc = 0.00mg, Speed = 1.59m/s, Dis = 1.65m
```

~10cm displacement

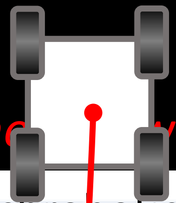


$$\ddot{s} = \dot{v} = a$$

# Accelerometer

- Determining tilt and roll √
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1*: Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off

*Errors only accumulate, and the[illegible] fast!*



www.chrobotics.com

| Angle Error (degrees) | Acceleration Error (m/s/s) | Velocity Error (m/s) at 10 seconds | Position Error (m) at 10 seconds | Position Error (m) at 1 minute | Position Error (m) at 10 minutes | Position Error (m) at 1 hour |
|---|---|---|---|---|---|---|
| 0.1 | 0.017 | 0.17 | 1.7 | 61.2 | 6120 | 220 e 3 |
| 0.5 | 0.086 | [illegible] | 8.6 | 309.6 | 30960 | 1.1 e 6 |
| 1.0 | 0.17 | [illegible] | 17 | [illegible] | [illegible]200 | 2.2 e 6 |
| 1.5 | 0.256 | [illegible] | 25.6 | 921.6 | 92160 | 3.3 e 6 |
| 2.0 | 0.342 | 3.4[illegible] | 34.2 | 1231.2 | 123120 | 4.4 e 6 |
| 3.0 | 0.513 | 5.13 | 51.3 | 1846.8 | 184680 | 6.6 e 6 |
| 5.0 | 0.854 | 8.54 | 85.4 | 3074.4 | 307440 | 11 e 6 |

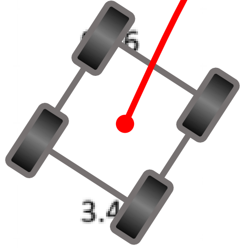
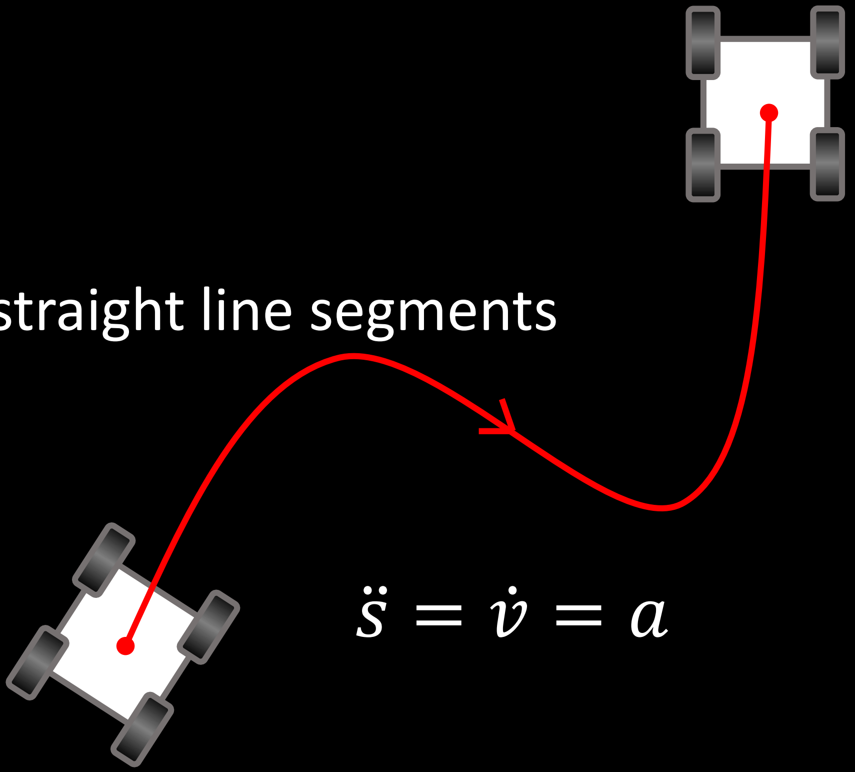


Table 1 - A summary of velocity and position errors caused by attitude estimation error.

# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1*: Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off
  - *Solution 4:* Stop periodically and zero the velocity
  - *Solution 5:* Use in combination with TOF sensor on straight line segments
  - *Solution 6:* Buy a more expensive IMU
  - etc…



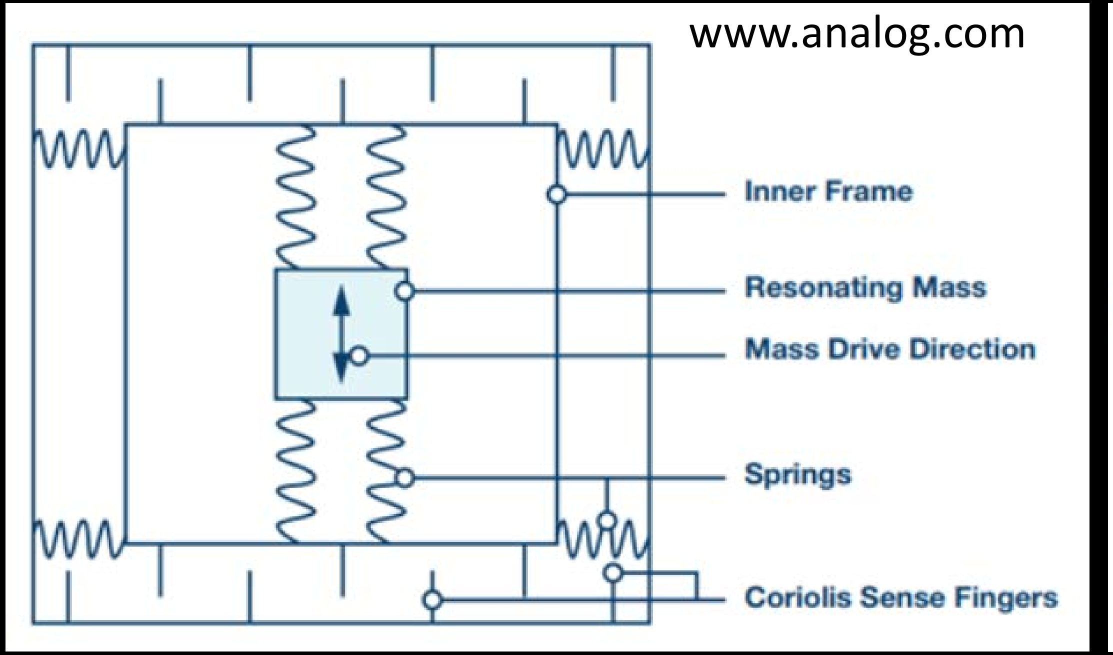
$$\ddot{s} = \dot{v} = a$$

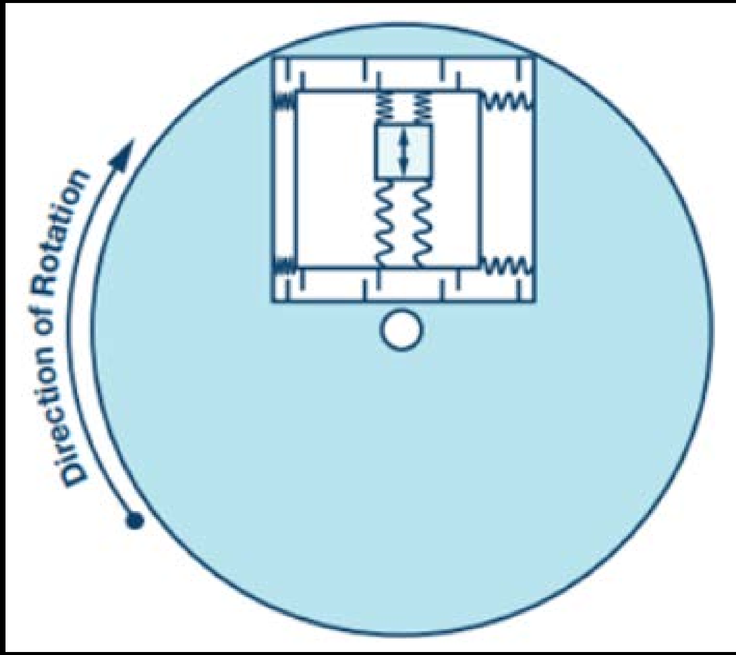
Prof. Kirstin Hagelskjær Petersen
kirstin@cornell.edu

# GYROSCOPE

# Gyroscopes

- Measures the rate of angular change [deg/s]



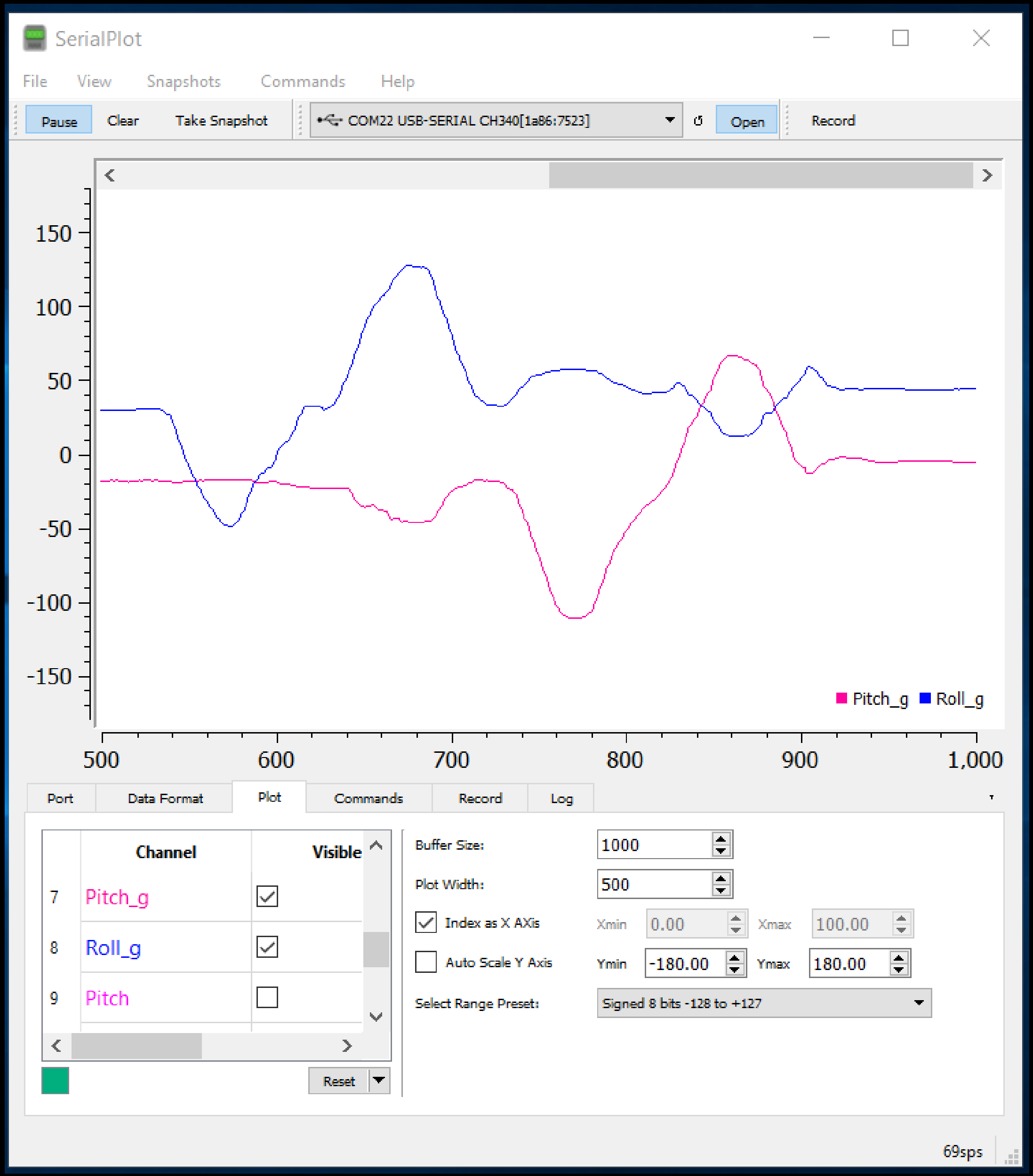



# Gyroscopes

- Measures the rate of angular change [deg/s]

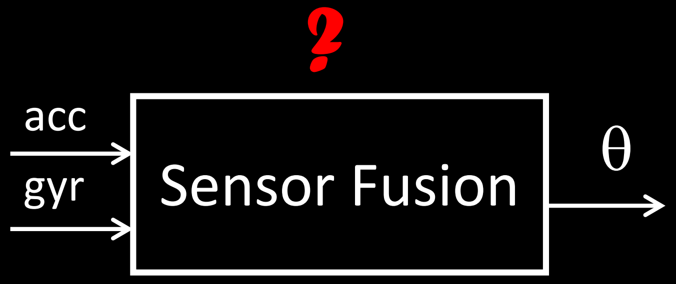
# Gyroscopes

- Measures the rate of angular change [deg/s]
- How to use the gyroscope to measure angles?
  - $\theta_g = \theta_g - gyr\_reading*dt$
- *Drift, but low noise*



# Gyroscopes

- Measures the rate of angular change [deg/s]
- How to use the gyroscope to measure angles?
  - $\theta_g = \theta_g - gyr\_reading * dt$
- *Drift, but low noise*
  - Complimentary to the accelerometer!
- Complimentary filter:
  - $\theta = (\theta + \theta_g * dt)(1-\alpha) + \theta_a \alpha$

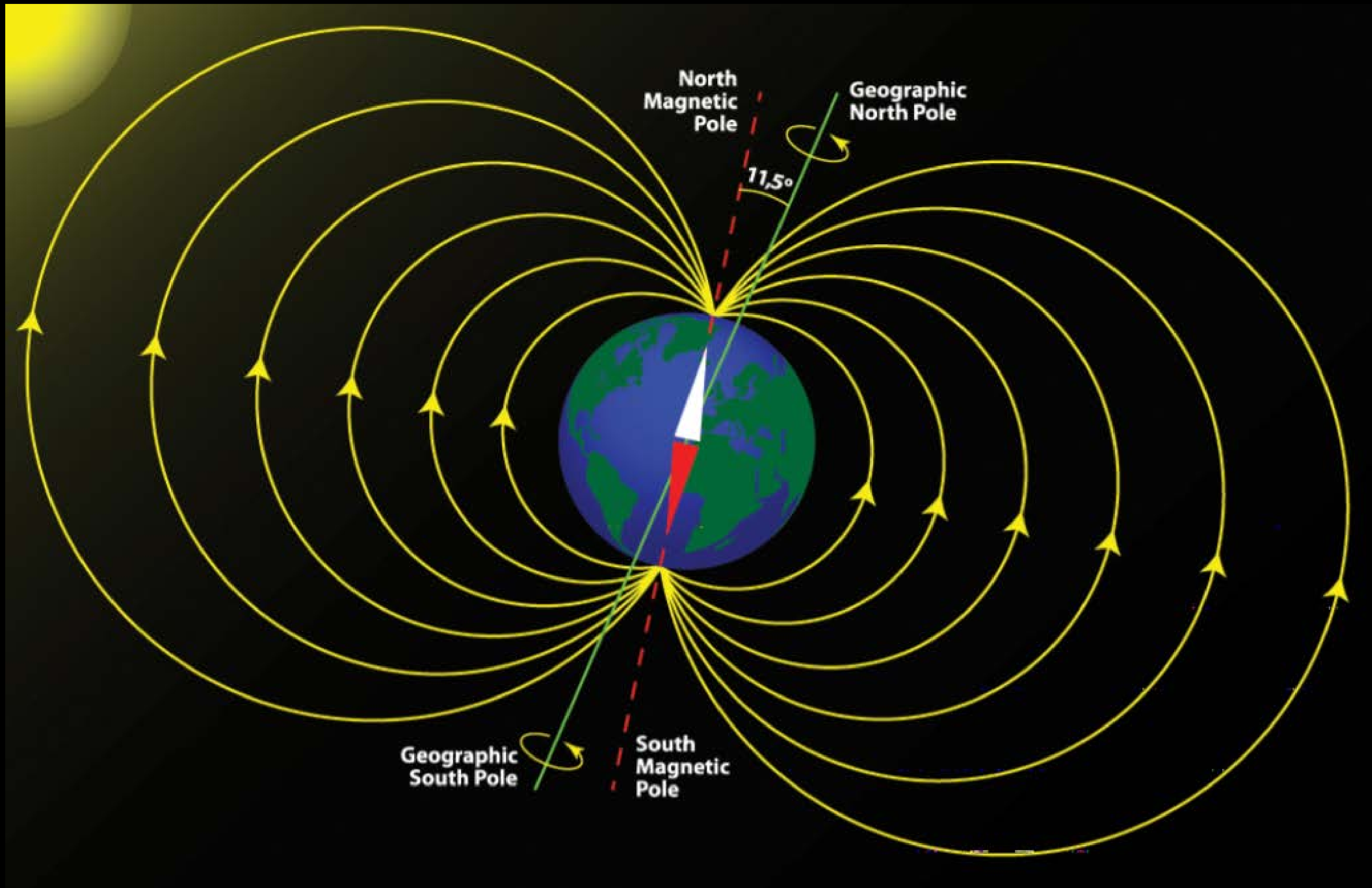



***Can we also estimate yaw?***

- Yes! (but there is no complementary data from the accelerometer)

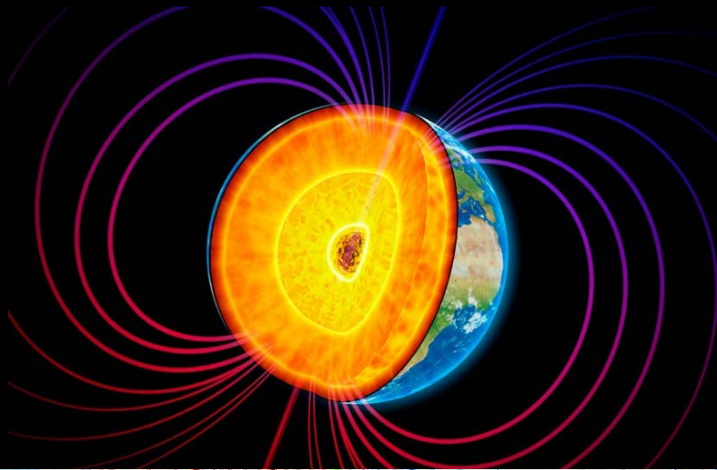
# Magnetometer

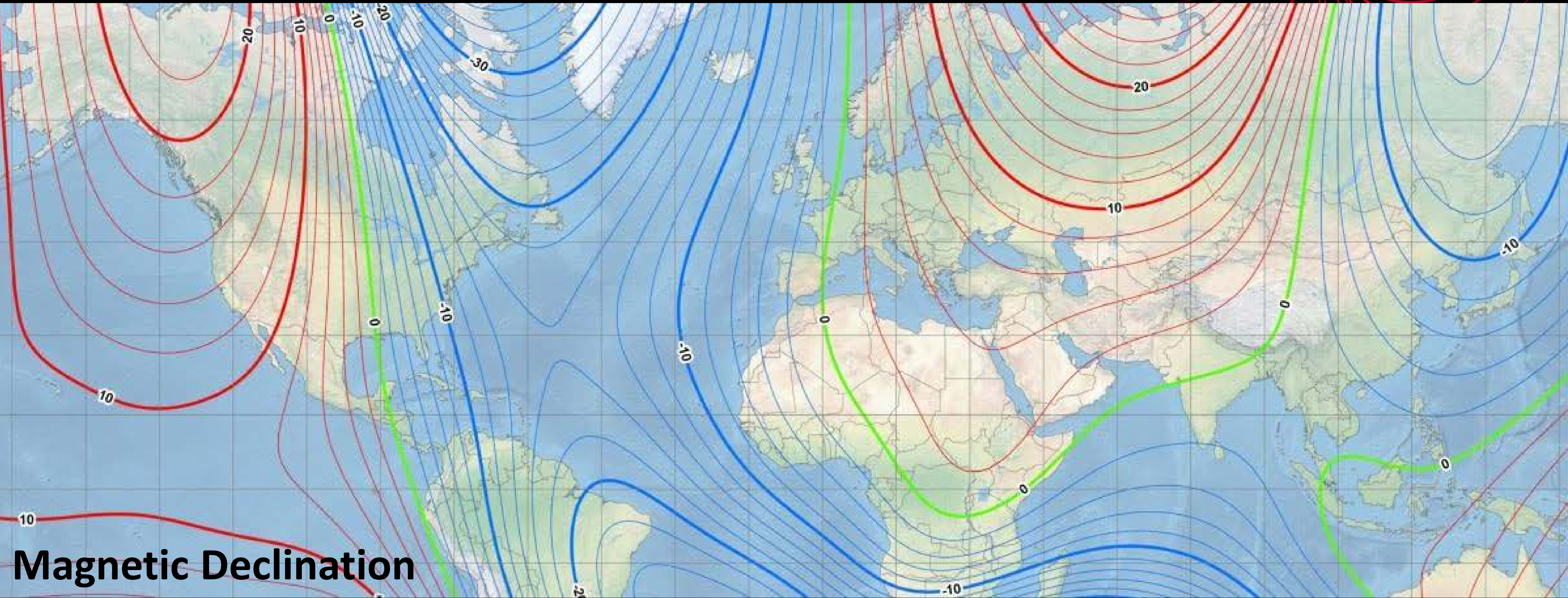
- Measure the Earth's magnetic field [Gauss] or [uT]
- The actual direction depends on latitude, longitude, and time



# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- The actual direction depends on latitude, longitude, and time
- Distortions due to metal objects or nearby EM fields





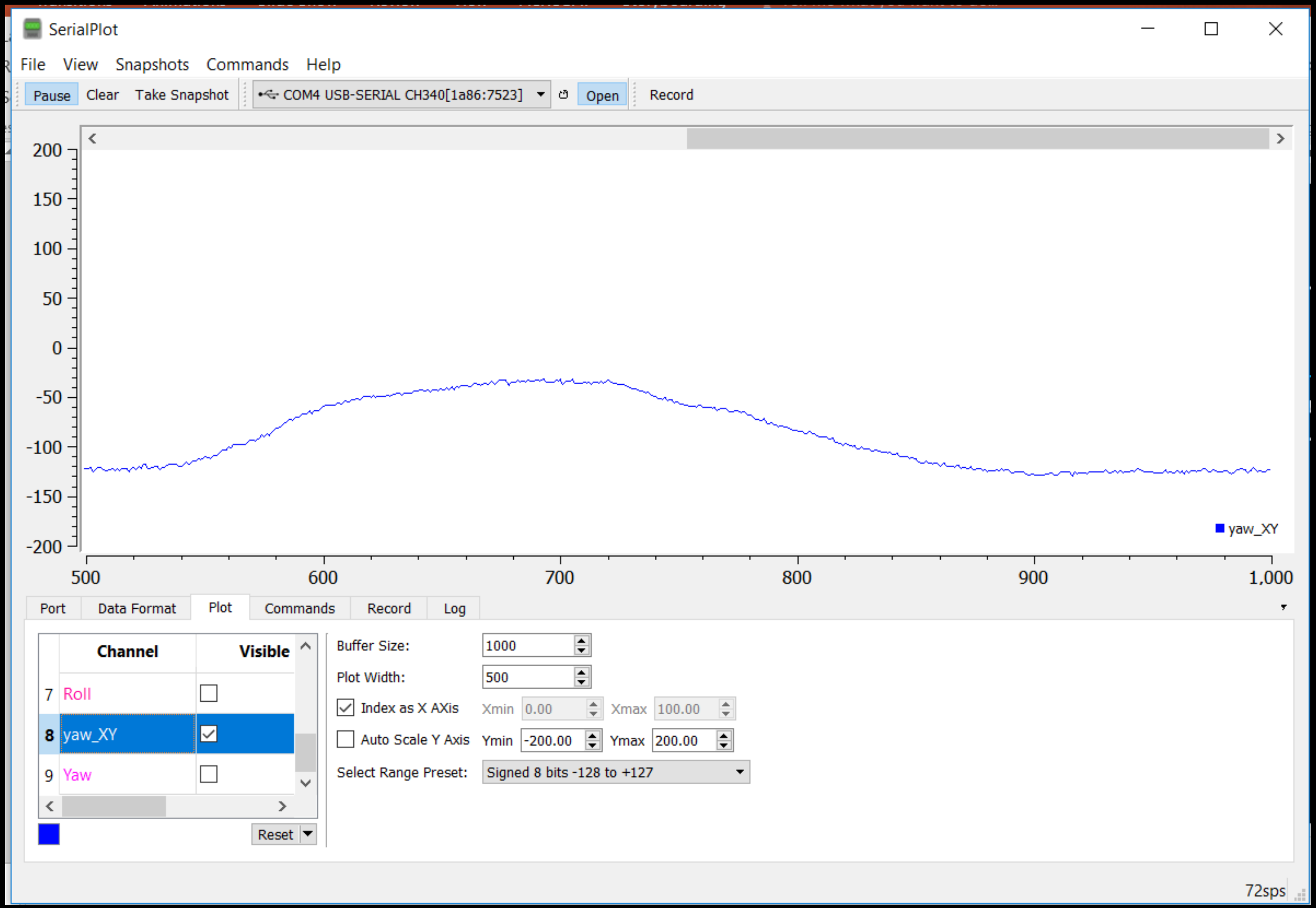

**Magnetic Declination**

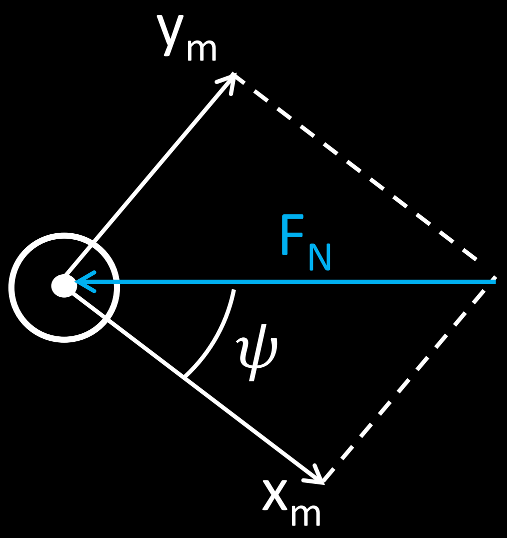
# Magnetometer

- Measure the Earth’s magnetic field [Gauss] or [uT]
- The actual direction depends on latitude, longitude, and time
- Distortions due to metal objects or nearby EM fields
    - ***Examples?***

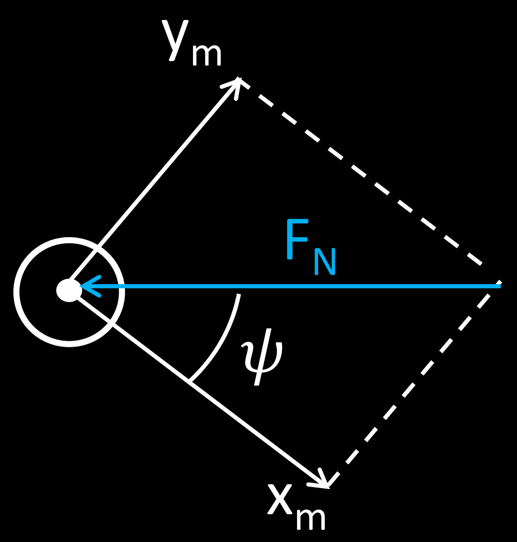
# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$



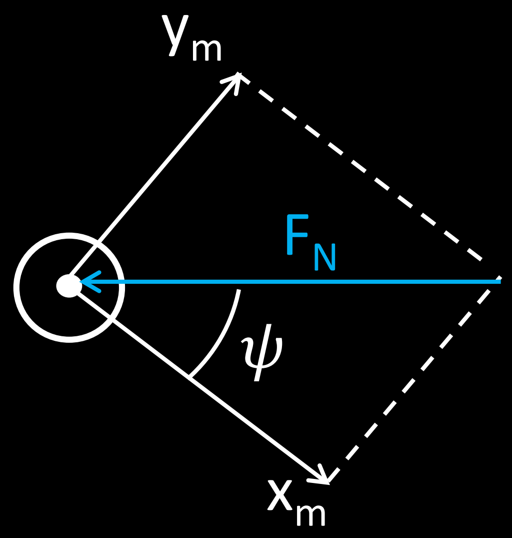
# Magnetometer

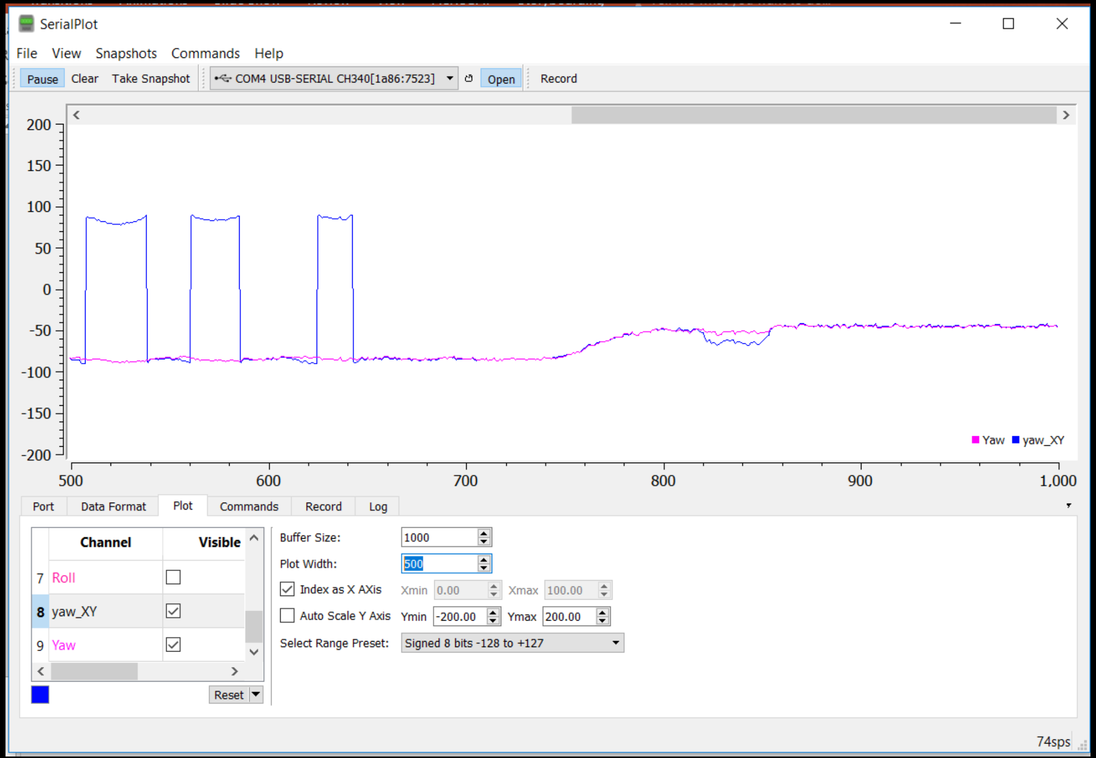



- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$
- What if you are also experiencing pitch and roll?
  - Fuse accelerometer + gyroscope + magnetometer data
- Tilt-compensated compass
  - $\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = R_{x,\phi} R_{y,\theta} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
  - $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = {R_{x,\phi}}^T {R_{y,\theta}}^T \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ \sin(\phi)\sin(\theta) & \cos(\phi) & \cos(\theta)\sin(\phi) \\ \cos(\phi)\sin(\theta) & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}$
  - x = $y_m$*cos($\phi$) - $z_m$*sin($\phi$);
  - y = $x_m$*cos(θ) + $y_m$*sin($\phi$)*sin(θ) + $z_m$*cos($\phi$)*sin(θ);
  - $\psi$ = atan2(y,x)

# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$
- What if you are also experiencing pitch and roll?
  - Fuse accelerometer + gyroscope + magnetometer data
- Tilt-compensated compass



# Lab 3: Characterize your Car

- Open ended…
  - But we expect *useful* data
  - Qualitative and quantitative analysis
  - Structured experiments
  - Proof and replicas
- What would be helpful to know?

Measure

  - Dimension (chassis/wheels)
  - Mass
  - Battery life time

Experimental

  - Static friction
  - Braking distance
  - Velocity range / motor supply voltage
  - Acceleration range
  - Tricks

***How would you set this up?***
***How would you measure this?***

- Feel free to work in teams (remember to give credits)
- Set aside time every day
- Deliverable: Github page and 3-5 min presentation 9/29th

# Sources and References


- http://www.chrobotics.com/library/accel-position-velocity
- EE 267 Virtual Reality, by Gordon Wetzstein at Stanford University
- Analog.com
- https://toptechboy.com/