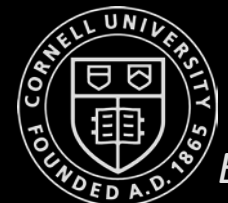
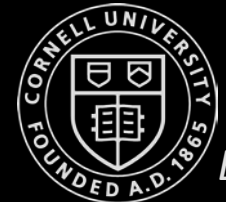


# Fast Robots



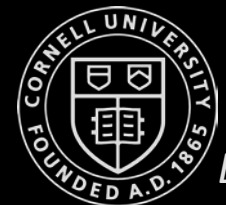
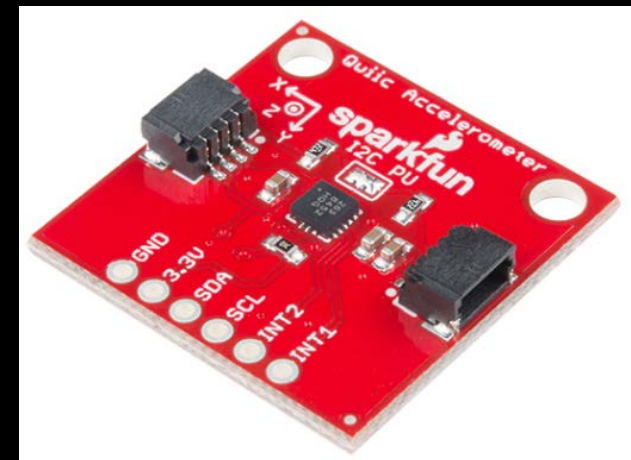
# IMU

- Inertial Measurement Unit
  - Data related to orientation, velocity, and gravity



# IMU

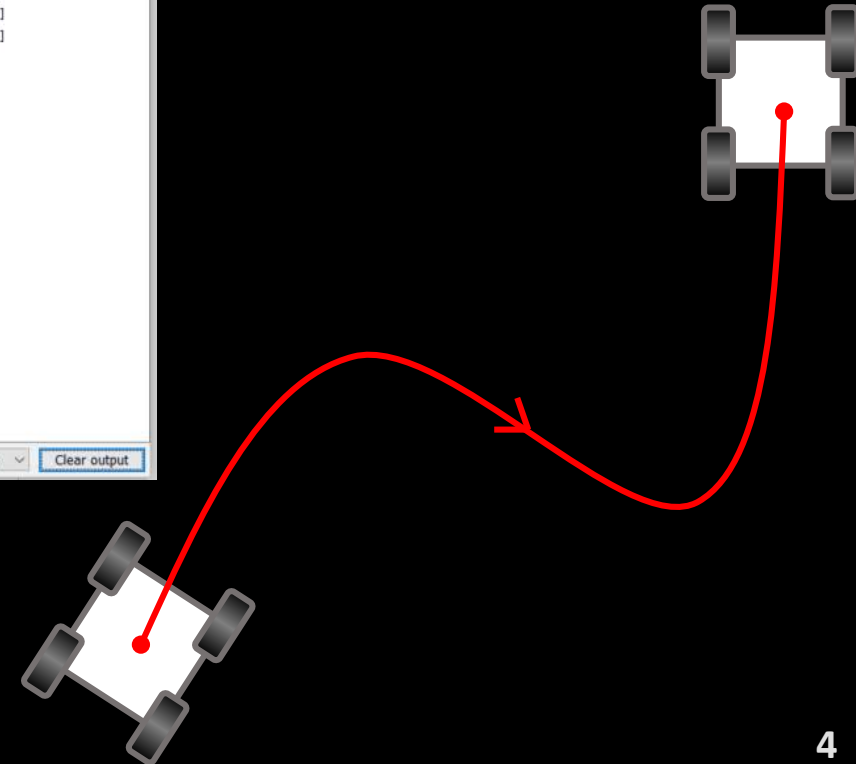
- Inertial Measurement Unit
- *Accelerometer*
  - Linear acceleration,  $a = \dot{v}$  [m/s<sup>2</sup>] → Track orientation (position)
- *Gyroscope*
  - Angular velocity,  $\dot{\omega} = \frac{\Delta\theta}{\Delta t}$  [deg/sec] → Track orientation
- *Magnetometer*
  - Magnetic field strength, [uT] or [Gauss], (1 Gauss = 100uT) → Get absolute orientation
- *NB: Gravity, magnetic fields, accelerations affect these sensors in many ways!*



# IMU - Demo

- ..\SparkFun\_ICM-20948\_ArduinoLibrary-master\examples\Arduino\Example1\_Basics

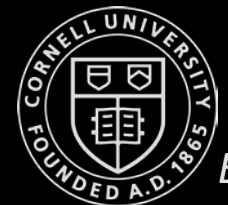
```
COM4
Initialization of the sensor returned: All is well.
Waiting for data
Scaled. Acc (mg) [ -00093.75, 00001.46, 01019.53 ], Gyr (DPS) [ -00000.96, 00001.80, -00002.67 ], Mag (uT) [ 00001.05, -00049.95, 00049.50 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00090.82, 00010.74, 01012.21 ], Gyr (DPS) [ 00001.40, 00000.82, 00001.05 ], Mag (uT) [ 00002.10, -00050.10, 00049.05 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00089.84, 00001.46, 01025.39 ], Gyr (DPS) [ 00001.19, 00000.60, 00002.05 ], Mag (uT) [ 00001.95, -00049.95, 00049.95 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00104.00, 00007.32, 01018.07 ], Gyr (DPS) [ -00001.53, 00001.66, -00002.59 ], Mag (uT) [ 00002.70, -00051.45, 00048.75 ], Tmp (C) [ 00024.07 ]
Scaled. Acc (mg) [ -00087.89, -00003.91, 01010.74 ], Gyr (DPS) [ -00000.18, 00001.04, 00001.18 ], Mag (uT) [ 00001.50, -00050.40, 00049.20 ], Tmp (C) [ 00024.16 ]
Scaled. Acc (mg) [ -00087.89, -00004.39, 01024.90 ], Gyr (DPS) [ 00003.80, -00001.62, -00000.11 ], Mag (uT) [ 00001.95, -00050.70, 00050.70 ], Tmp (C) [ 00024.26 ]
Scaled. Acc (mg) [ -00096.19, 00007.32, 01017.09 ], Gyr (DPS) [ 00000.19, 00002.37, -00002.16 ], Mag (uT) [ 00002.10, -00050.55, 00049.05 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00089.36, -00002.44, 01021.97 ], Gyr (DPS) [ 00000.73, -00000.73, 00004.83 ], Mag (uT) [ 00003.30, -00050.10, 00050.10 ], Tmp (C) [ 00024.40 ]
Scaled. Acc (mg) [ -00100.59, -00002.93, 01012.21 ], Gyr (DPS) [ 00001.35, 00000.65, 00001.63 ], Mag (uT) [ 00002.25, -00050.70, 00049.95 ], Tmp (C) [ 00024.07 ]
Scaled. Acc (mg) [ -00103.52, -00001.46, 01014.16 ], Gyr (DPS) [ -00000.80, 00001.38, -00004.44 ], Mag (uT) [ 00001.05, -00050.40, 00049.20 ], Tmp (C) [ 00024.35 ]
Scaled. Acc (mg) [ -00095.21, -00000.49, 01015.14 ], Gyr (DPS) [ 00000.66, -00000.41, 00001.28 ], Mag (uT) [ 00001.95, -00051.00, 00049.20 ], Tmp (C) [ 00024.45 ]
```



**ECE 4960**

Prof. Kirstin Hagelskjær Petersen  
kirstin@cornell.edu

# ACCELEROMETER



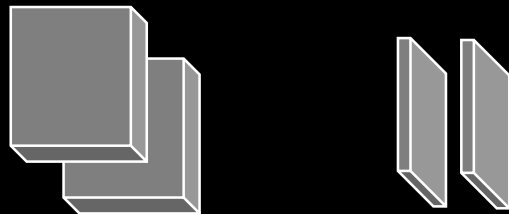
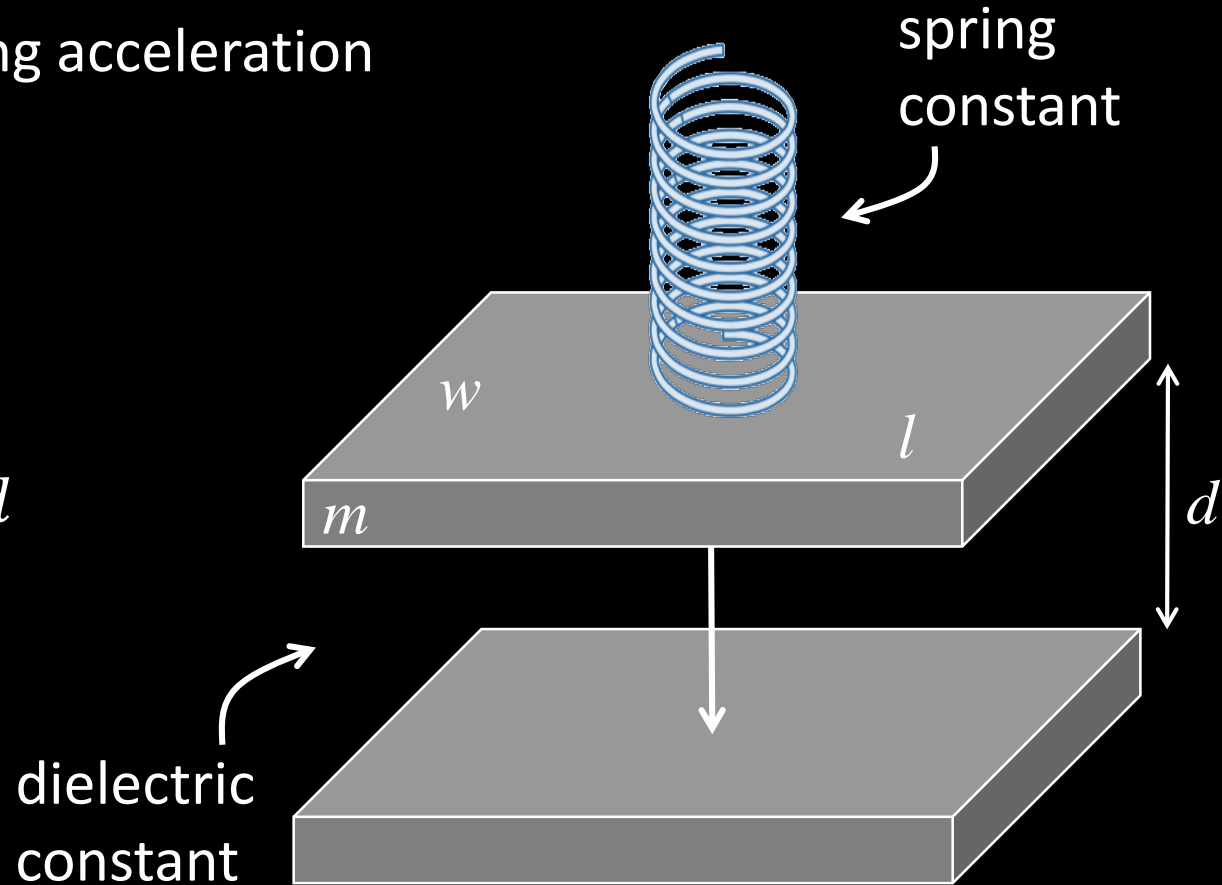
# Accelerometer

- Measuring acceleration

$$C = \frac{\epsilon A}{d}$$

$$F = k\Delta d$$

$$F = ma$$



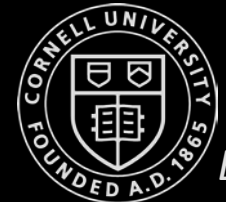
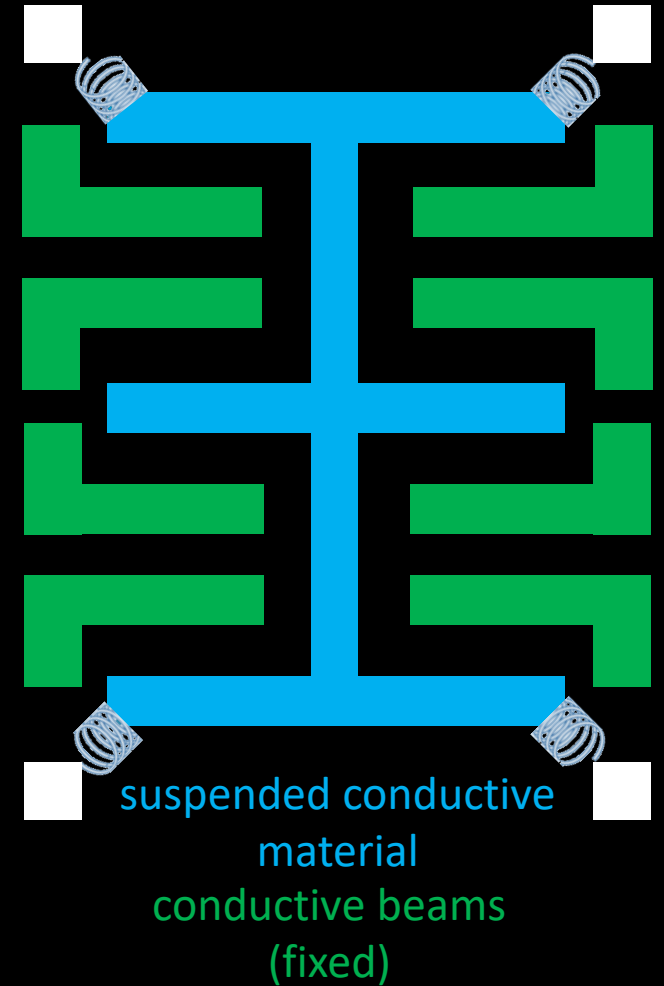
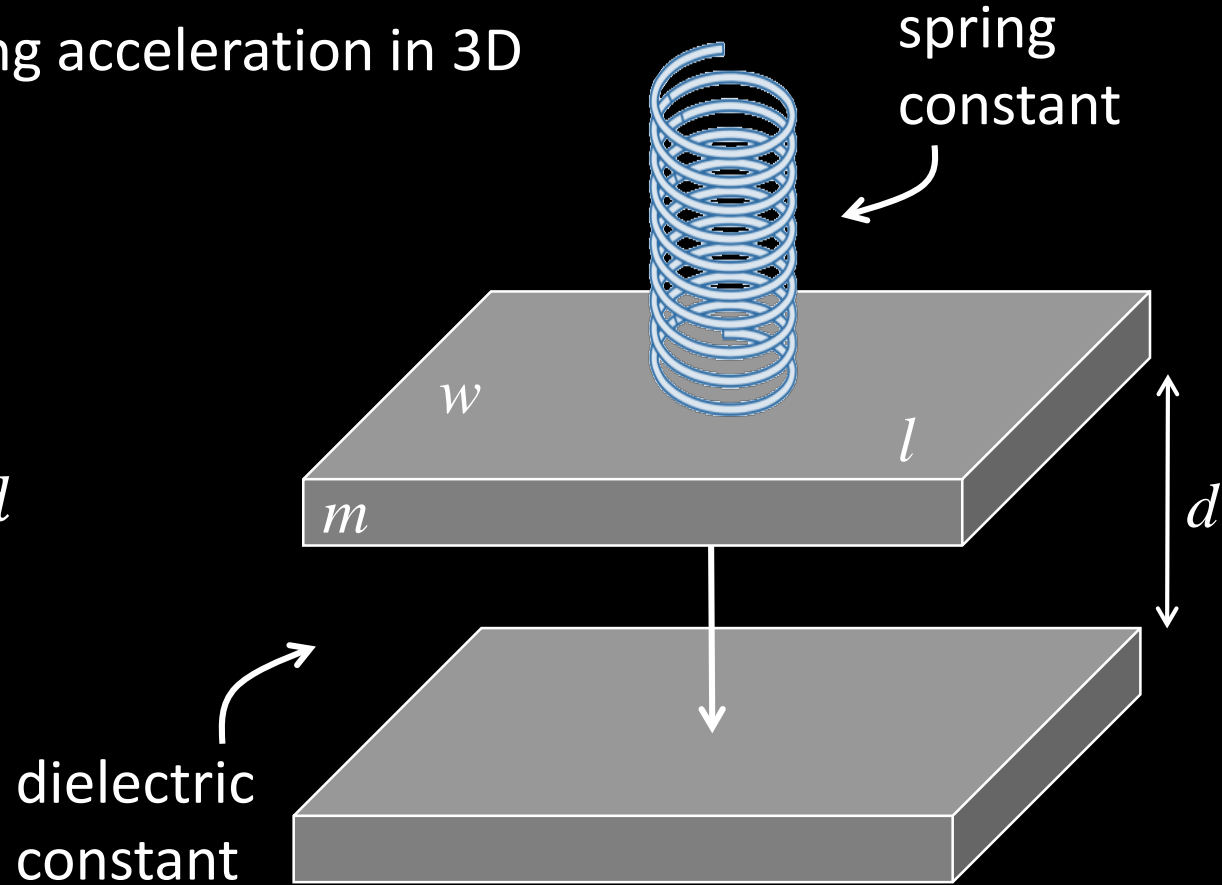
# Accelerometer

- Measuring acceleration in 3D

$$C = \frac{\epsilon A}{d}$$

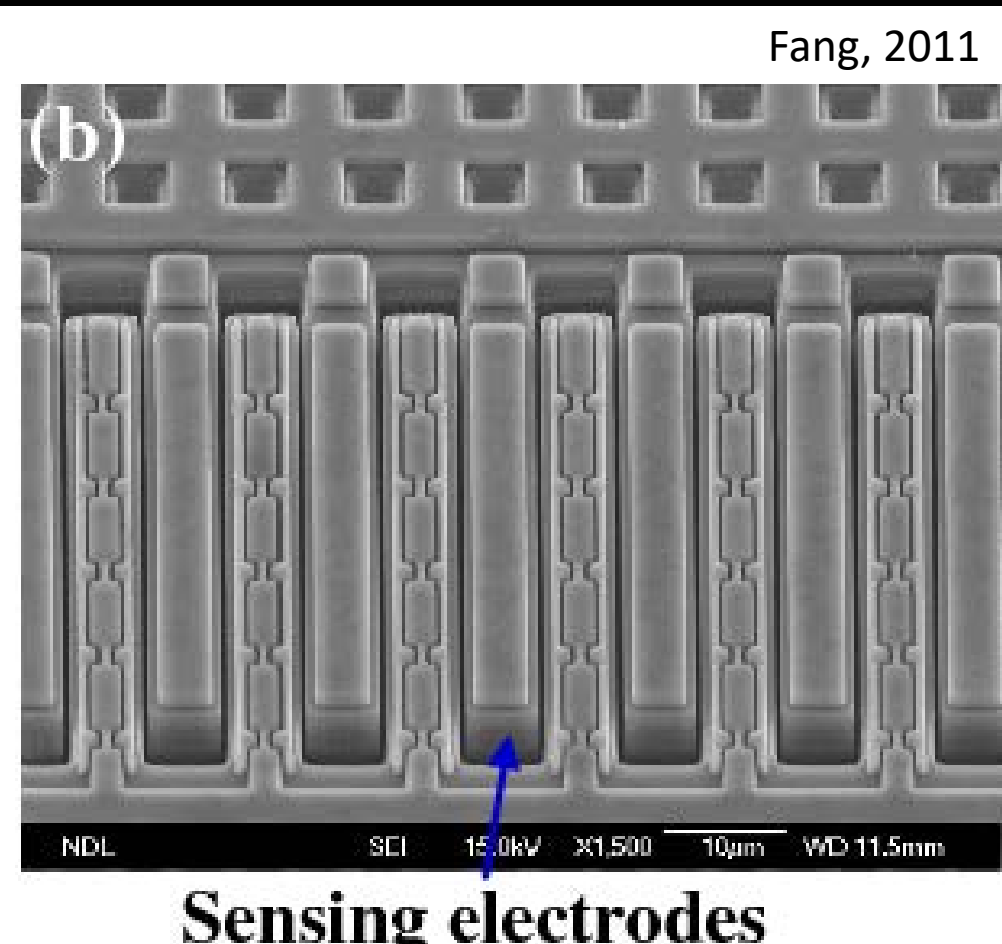
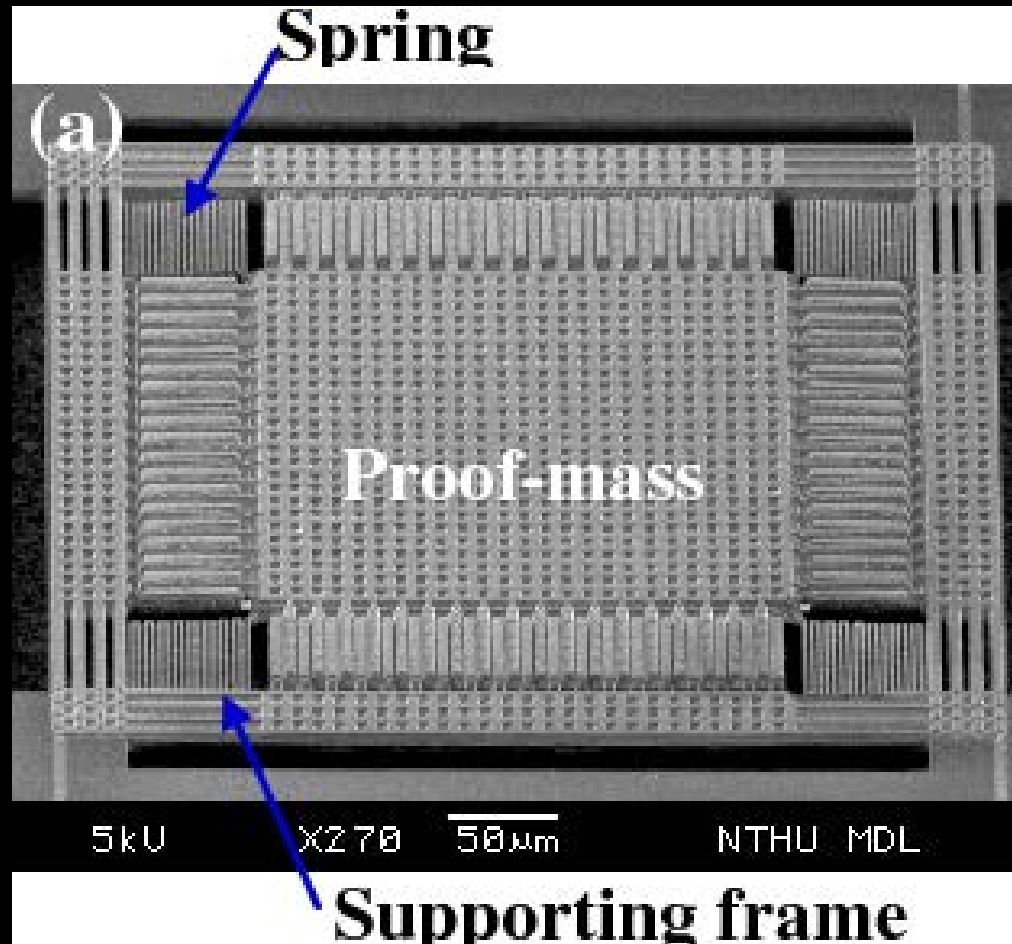
$$F = k\Delta d$$

$$F = ma$$



# Accelerometer

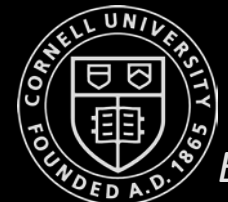
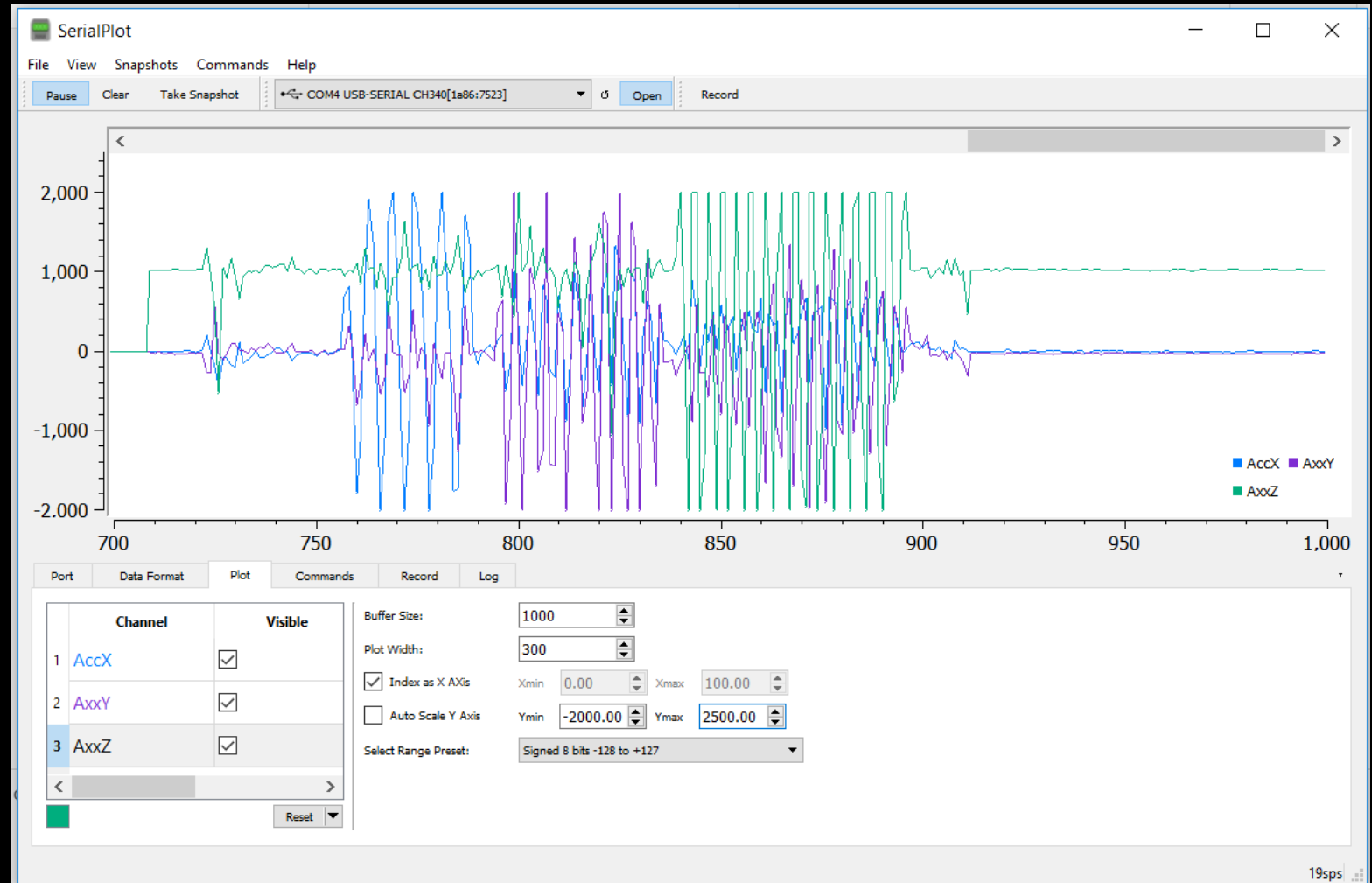
- Measuring acceleration in 3D
- Micro-Electro-Mechanical Systems





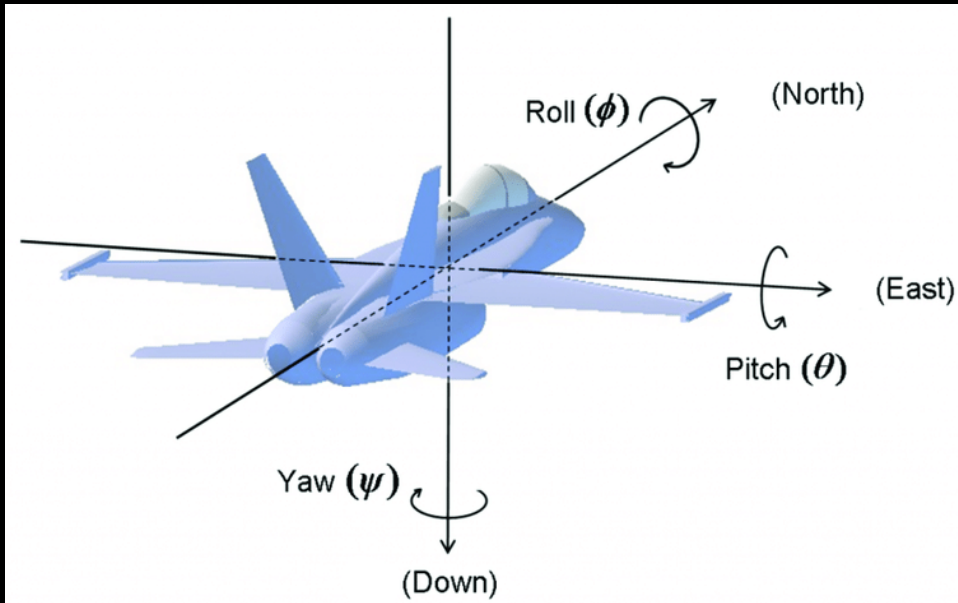
# Accelerometer

- Measuring acceleration in 3D
- Use a program like SerialPlot to visualize your data

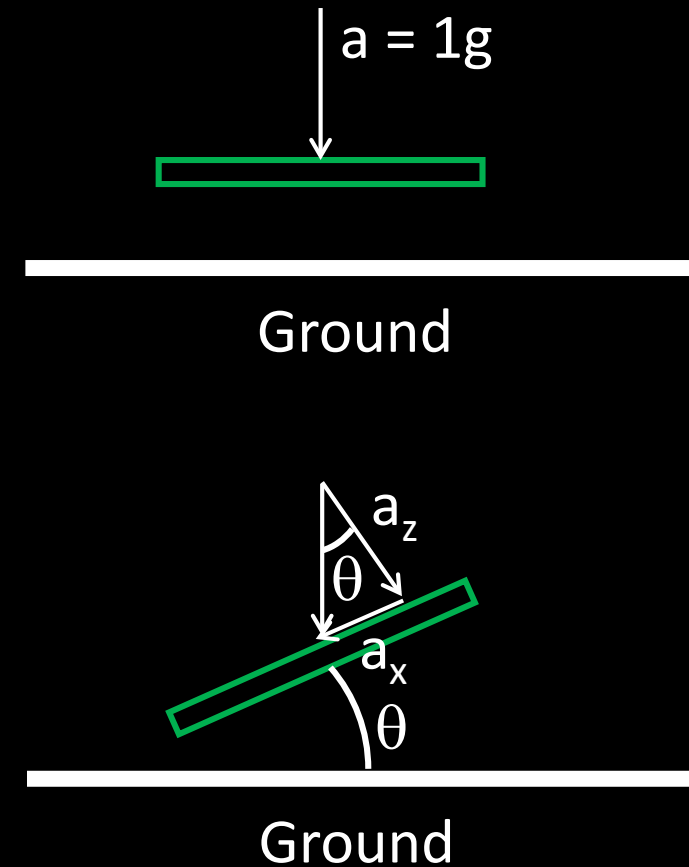


# Accelerometer

- How to use the accelerometer to determine roll, tilt, and ~~yaw?~~



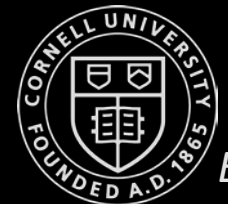
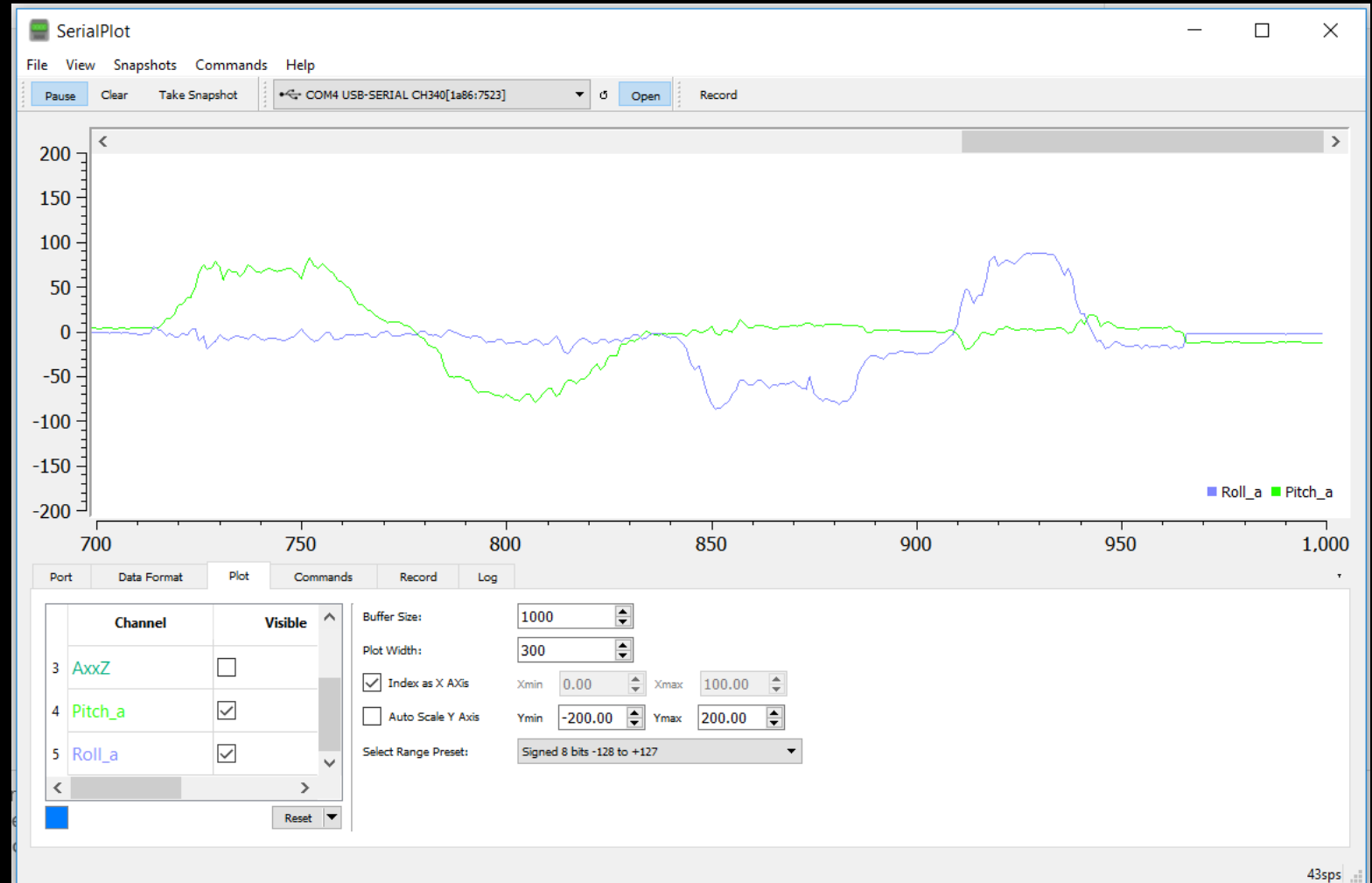
- $a_x = 1g \sin \theta$
- $a_z = 1g \cos \theta$
- $a_x / a_z = \tan \theta$
- $\theta = \text{atan}(a_x/a_z)$       •  $\phi = \text{atan}(a_y/a_z)$ 
  - Remember, use atan2!



*How do you measure yaw  
with the accelerometer?*

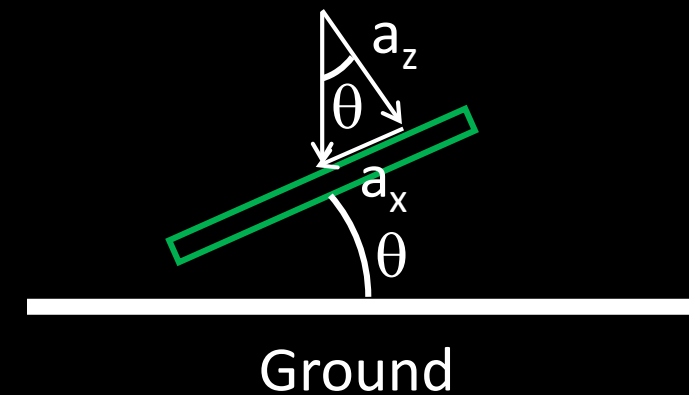
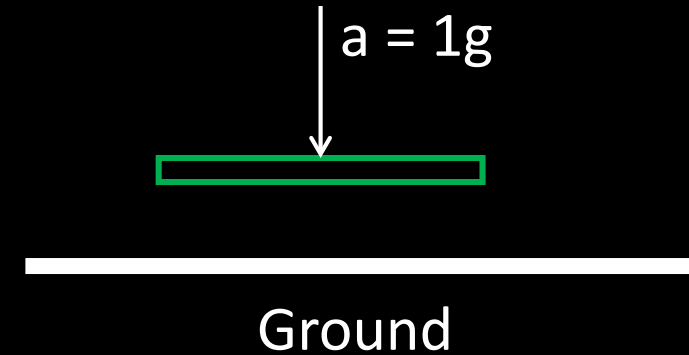
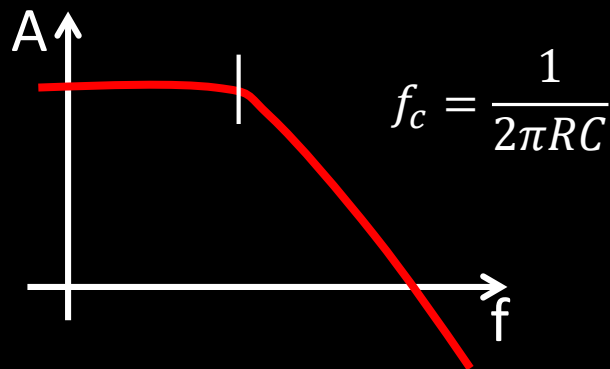
# Accelerometer

- Determining tilt and roll
- $\theta = \text{atan}(a_x/a_z)$
- $\phi = \text{atan}(a_y/a_z)$



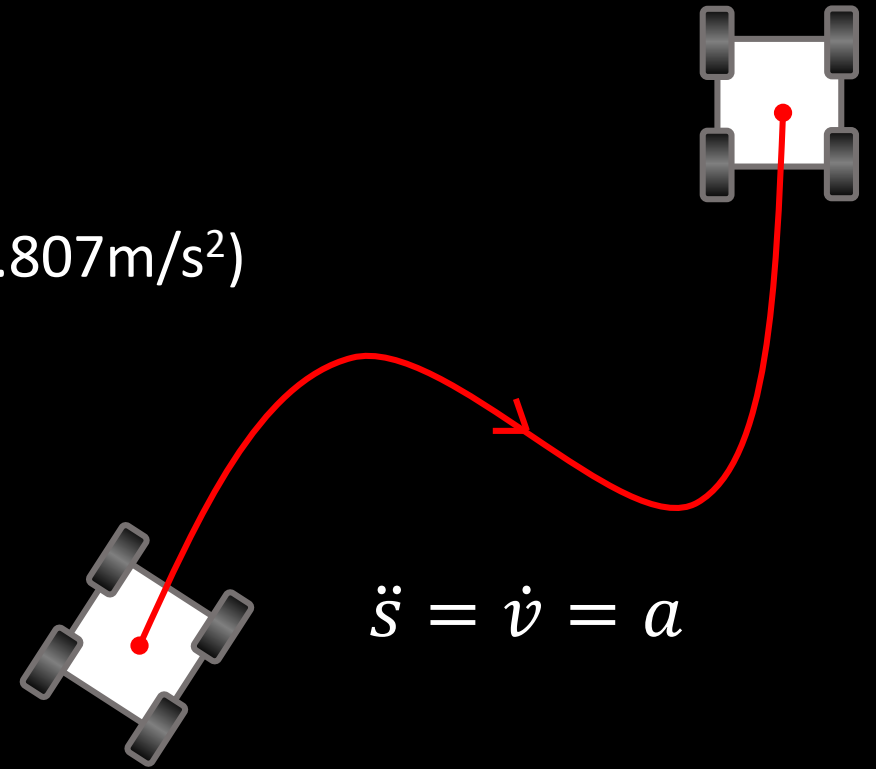
# Accelerometer

- Determining tilt and roll
- Good (very accurate on average) vs Bad (noisy)
- Low pass complimentary filter
  - $\theta_{\text{LPF}}[n] = \alpha * \theta + (1 - \alpha) * \theta_{\text{LPF}}[n-1]$
  - $\theta_{\text{LPF}}[n-1] = \theta_{\text{LPF}}[n]$
  - $\alpha = \frac{T}{T+RC}$



# Accelerometer

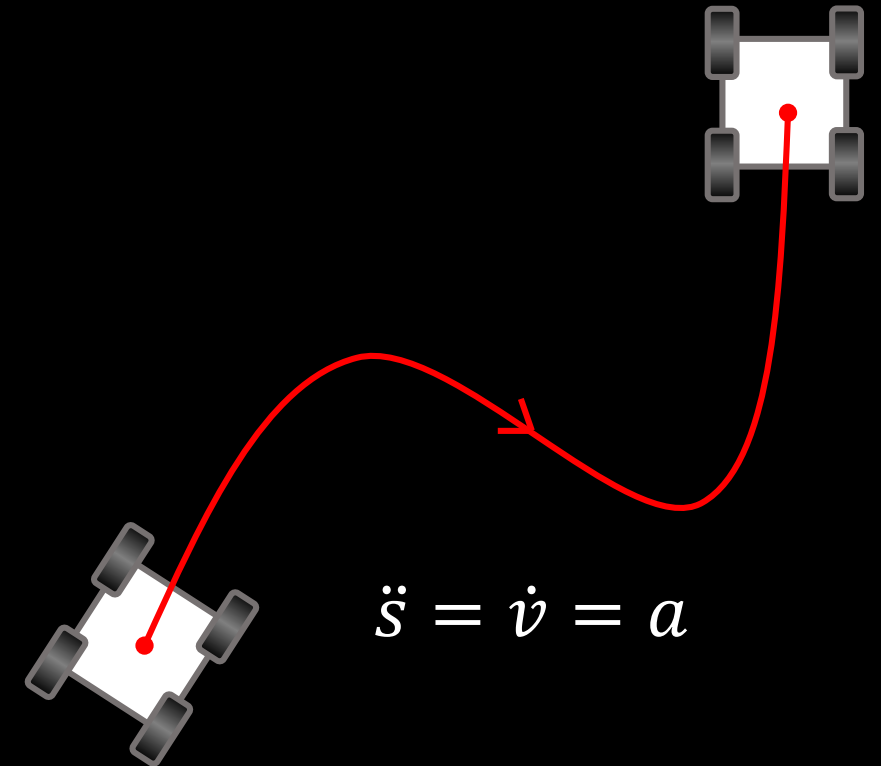
- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - $v = \int a$
  - $s = \int \int a$
  - $v[k+1] = v[k] + a[k] * dt$
  - $s[k+1] = s[k] + v[k] * dt$
- *Remember:* The accelerometer output is in mg ( $1g = 9.807m/s^2$ )



# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration

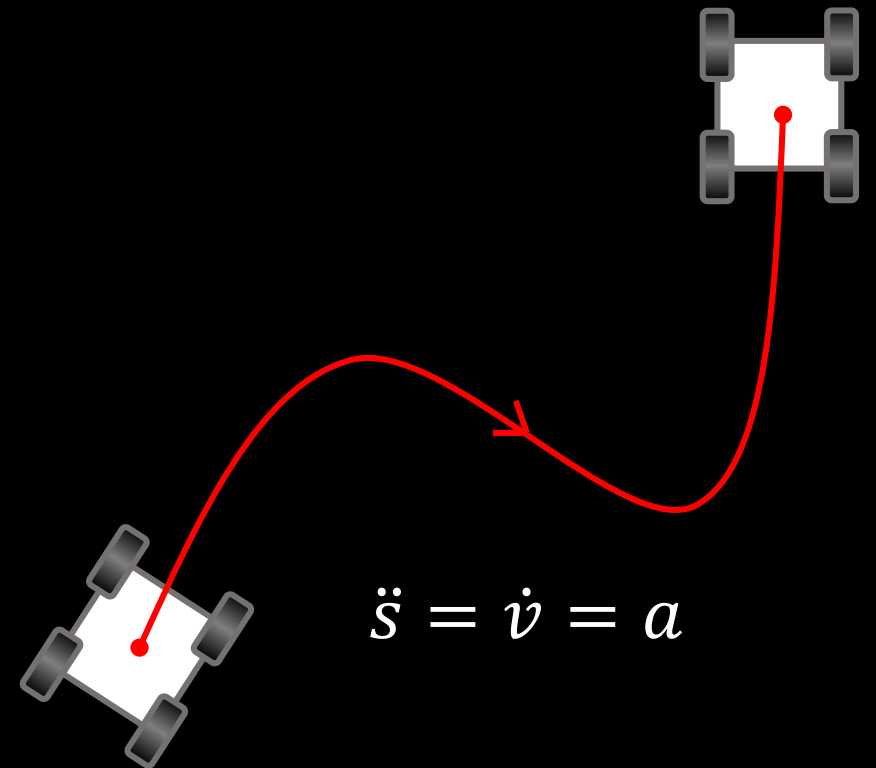
```
COM3
Waiting for data
dt= 0.01s, Acc = -0.01mg, Speed = -0.01m/s, Dis = -0.00m
dt= 0.01s, Acc = -0.04mg, Speed = -0.03m/s, Dis = -0.01m
dt= 0.01s, Acc = -0.15mg, Speed = -0.07m/s, Dis = -0.03m
dt= 0.01s, Acc = -0.15mg, Speed = -0.13m/s, Dis = -0.06m
dt= 0.01s, Acc = -0.17mg, Speed = -0.19m/s, Dis = -0.12m
dt= 0.01s, Acc = -0.17mg, Speed = -0.26m/s, Dis = -0.19m
dt= 0.01s, Acc = -0.18mg, Speed = -0.34m/s, Dis = -0.30m
dt= 0.01s, Acc = -0.21mg, Speed = -0.41m/s, Dis = -0.43m
dt= 0.01s, Acc = -0.22mg, Speed = -0.50m/s, Dis = -0.58m
dt= 0.01s, Acc = -0.25mg, Speed = -0.59m/s, Dis = -0.77m
dt= 0.01s, Acc = -0.27mg, Speed = -0.69m/s, Dis = -0.99m
dt= 0.01s, Acc = -0.28mg, Speed = -0.79m/s, Dis = -1.25m
dt= 0.01s, Acc = -0.29mg, Speed = -0.90m/s, Dis = -1.54m
dt= 0.01s, Acc = -0.31mg, Speed = -1.02m/s, Dis = -1.87m
dt= 0.01s, Acc = -0.32mg, Speed = -1.14m/s, Dis = -2.24m
dt= 0.01s, Acc = -0.34mg, Speed = -1.26m/s, Dis = -2.66m
dt= 0.01s, Acc = -0.35mg, Speed = -1.39m/s, Dis = -3.12m
dt= 0.01s, Acc = -0.36mg, Speed = -1.52m/s, Dis = -3.62m
dt= 0.01s, Acc = -0.35mg, Speed = -1.65m/s, Dis = -4.17m
dt= 0.01s, Acc = -0.36mg, Speed = -1.79m/s, Dis = -4.76m
dt= 0.01s, Acc = -0.36mg, Speed = -1.92m/s, Dis = -5.40m
dt= 0.01s, Acc = -0.34mg, Speed = -2.05m/s, Dis = -6.09m
dt= 0.01s, Acc = -0.35mg, Speed = -2.18m/s, Dis = -6.82m
dt= 0.01s, Acc = -0.37mg, Speed = -2.32m/s, Dis = -7.60m
dt= 0.01s, Acc = -0.38mg, Speed = -2.46m/s, Dis = -8.43m
```



# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1:* Calibrate the offset

```
COM4
Initialization of the sensor returned: All is well.
dt= 0.27s, Acc = -0.00mg, Speed = 0.56m/s, Dis = 0.09m
dt= 0.01s, Acc = -0.01mg, Speed = 0.56m/s, Dis = 0.28m
dt= 0.01s, Acc = 0.01mg, Speed = 0.56m/s, Dis = 0.47m
dt= 0.01s, Acc = 0.00mg, Speed = 0.56m/s, Dis = 0.66m
dt= 0.01s, Acc = -0.01mg, Speed = 0.56m/s, Dis = 0.85m
dt= 0.01s, Acc = 0.01mg, Speed = 0.56m/s, Dis = 1.04m
dt= 0.01s, Acc = 0.00mg, Speed = 0.56m/s, Dis = 1.23m
dt= 0.01s, Acc = -0.01mg, Speed = 0.56m/s, Dis = 1.42m
dt= 0.01s, Acc = 0.01mg, Speed = 0.56m/s, Dis = 1.61m
dt= 0.01s, Acc = 0.00mg, Speed = 0.56m/s, Dis = 1.80m
dt= 0.01s, Acc = -0.00mg, Speed = 0.56m/s, Dis = 1.99m
dt= 0.01s, Acc = -0.00mg, Speed = 0.56m/s, Dis = 2.18m
dt= 0.01s, Acc = -0.02mg, Speed = 0.55m/s, Dis = 2.37m
dt= 0.01s, Acc = -0.00mg, Speed = 0.55m/s, Dis = 2.55m
dt= 0.01s, Acc = -0.02mg, Speed = 0.55m/s, Dis = 2.74m
dt= 0.01s, Acc = -0.00mg, Speed = 0.54m/s, Dis = 2.93m
dt= 0.01s, Acc = -0.02mg, Speed = 0.54m/s, Dis = 3.11m
dt= 0.01s, Acc = -0.01mg, Speed = 0.54m/s, Dis = 3.29m
dt= 0.01s, Acc = 0.00mg, Speed = 0.53m/s, Dis = 3.47m
dt= 0.01s, Acc = -0.01mg, Speed = 0.53m/s, Dis = 3.66m
```

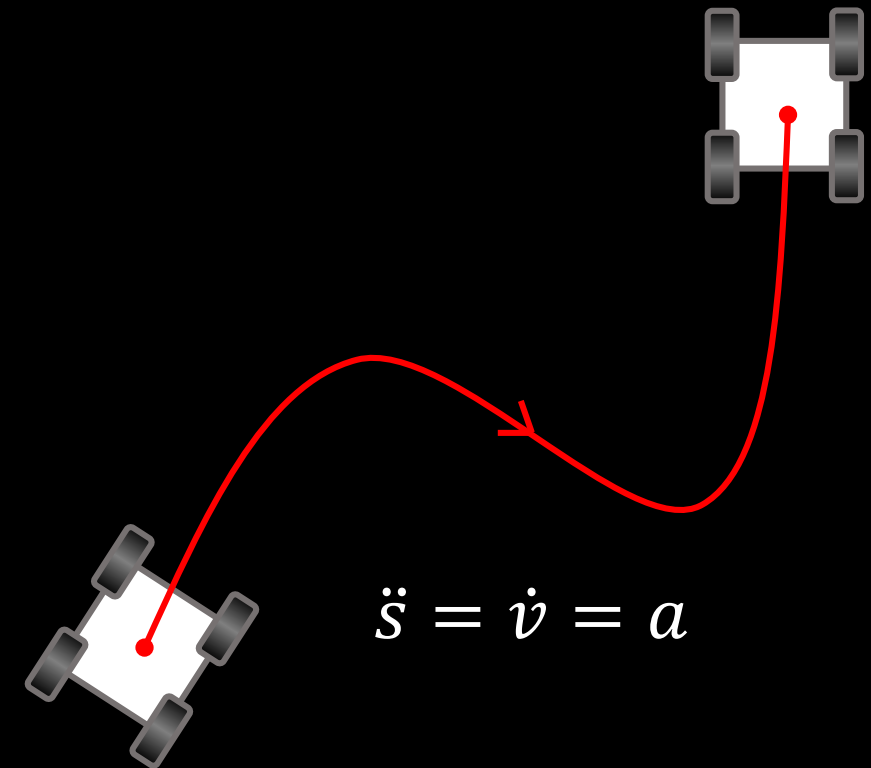


# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1:* Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off

```
COM4
dt= 0.27s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.00m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.01m/s, Dis = 0.00m
dt= 0.01s, Acc = 0.00mg, Speed = 0.01m/s, Dis = 0.00m
dt= 0.01s, Acc = 4.40mg, Speed = 0.93m/s, Dis = 0.12m ← ~10cm displacement
dt= 0.01s, Acc = 0.24mg, Speed = 1.54m/s, Dis = 0.58m
dt= 0.01s, Acc = 0.01mg, Speed = 1.58m/s, Dis = 1.11m
dt= 0.01s, Acc = 0.00mg, Speed = 1.59m/s, Dis = 1.65m
```

Autoscroll Show timestamp Newline 115200 baud Clear output

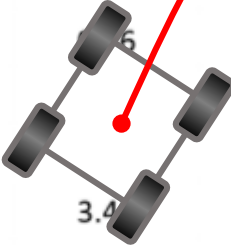




# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1:* Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off

Errors only accumulate, and they do so very fast!

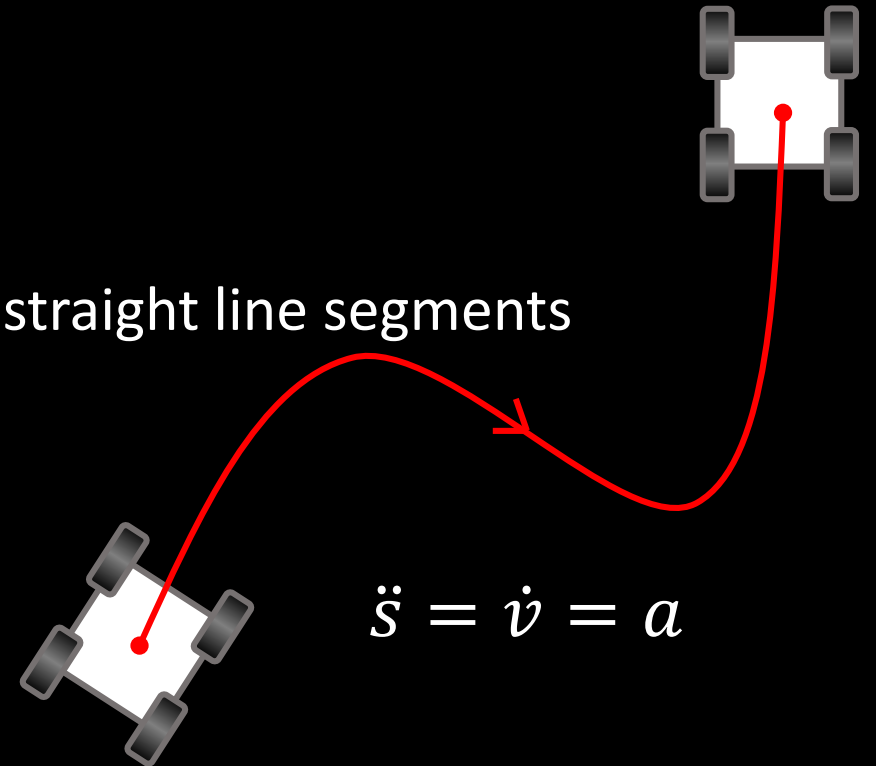


Angle Error (degrees)	Acceleration Error (m/s/s)	Velocity Error (m/s) at 10 seconds	Position Error (m) at 10 seconds	Position Error (m) at 1 minute	Position Error (m) at 10 minutes	Position Error (m) at 1 hour
0.1	0.017	0.17	1.7	61.2	6120	220 e 3
0.5	0.086	0.86	8.6	309.6	30960	1.1 e 6
1.0	0.17	1.7	17	619.2	61920	2.2 e 6
1.5	0.256	2.56	25.6	928.8	92880	3.3 e 6
2.0	0.342	3.42	34.2	1238.4	123840	4.4 e 6
3.0	0.513	5.13	51.3	1857.6	185760	6.6 e 6
5.0	0.854	8.54	85.4	3074.4	307440	11 e 6

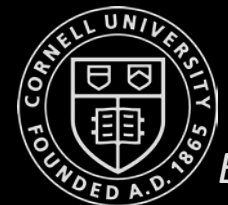
Table 1 - A summary of velocity and position errors caused by attitude estimation error.

# Accelerometer

- Determining tilt and roll ✓
- How to use the accelerometer to do dead reckoning?
  - *Issue:* Distinguishing acceleration of the sensor from gravitational acceleration
  - *Solution 1:* Calibrate the offset
  - *Solution 2:* Low pass filter the output
  - *Solution 3:* Minimum signal cut-off
  - *Solution 4:* Stop periodically and zero the velocity
  - *Solution 5:* Use in combination with TOF sensor on straight line segments
  - *Solution 6:* Buy a more expensive IMU
  - etc...

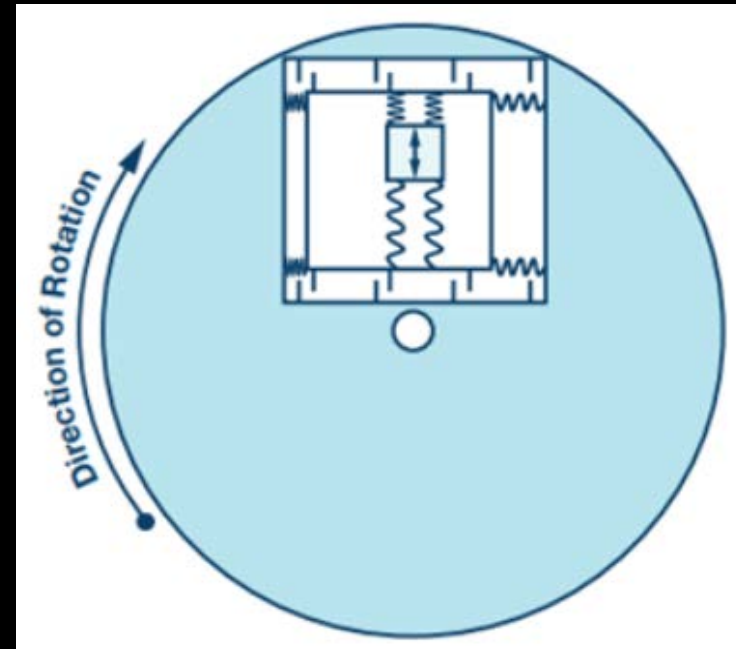
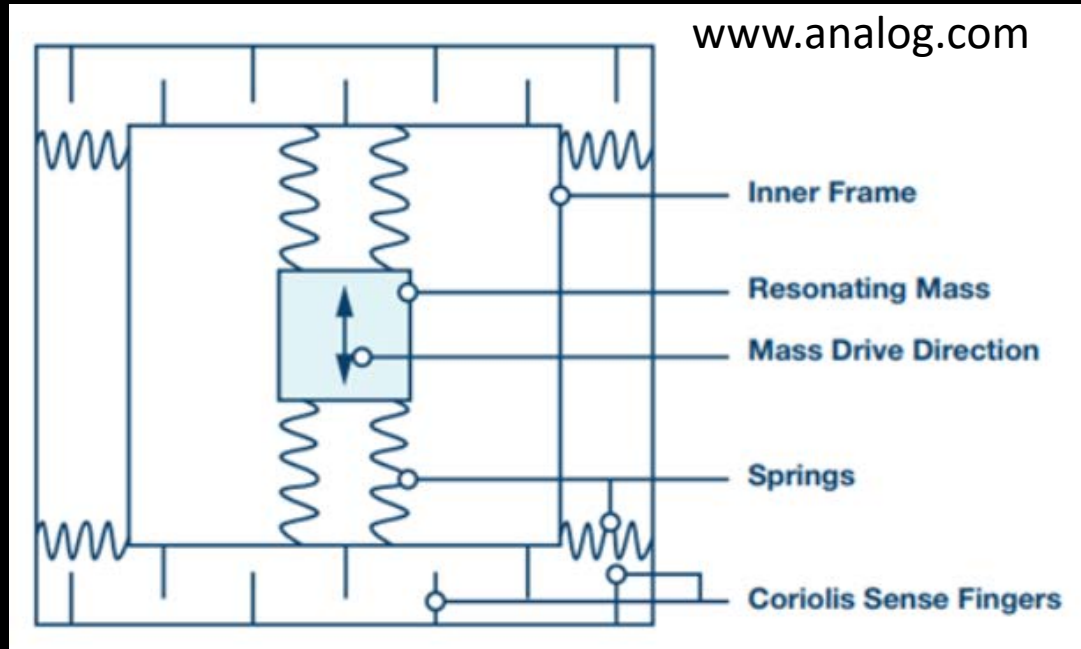


# GYROSCOPE



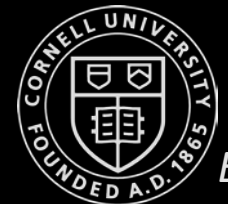
# Gyroscopes

- Measures the rate of angular change [deg/s]



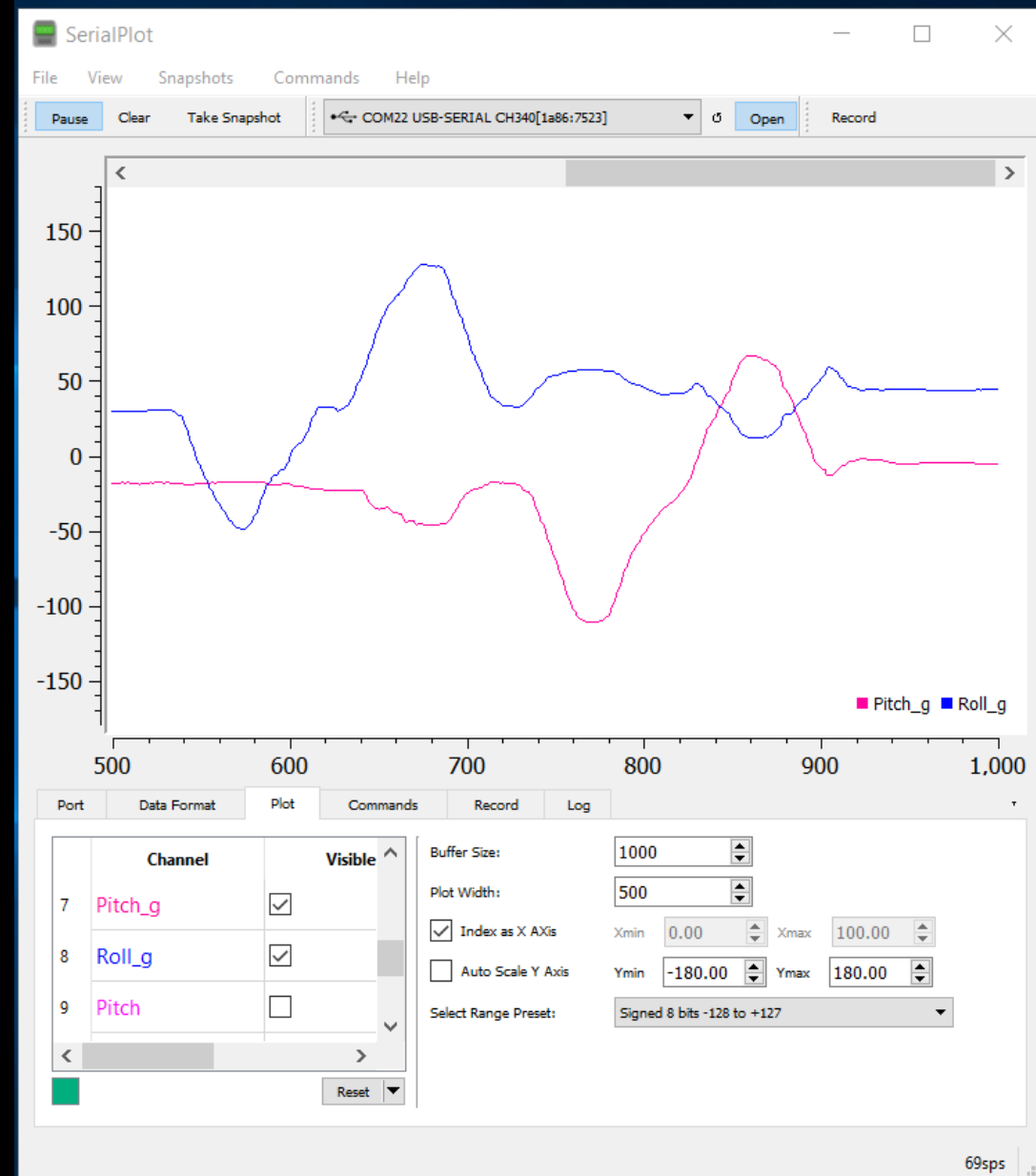
# Gyroscopes

- Measures the rate of angular change [deg/s]



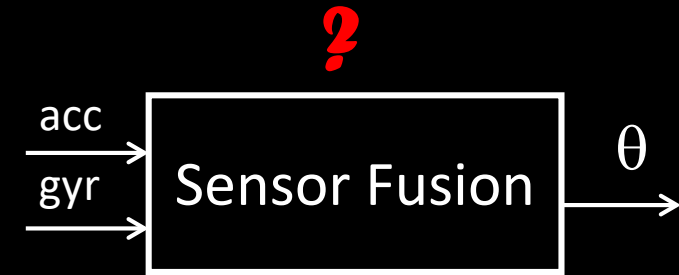
# Gyroscopes

- Measures the rate of angular change [deg/s]
- How to use the gyroscope to measure angles?
  - $\theta_g = \theta_g - \text{gyr\_reading} * dt$
- *Drift, but low noise*



# Gyroscopes

- Measures the rate of angular change [deg/s]
- How to use the gyroscope to measure angles?
  - $\theta_g = \theta_g - \text{gyr\_reading} * dt$
- *Drift, but low noise*
  - Complimentary to the accelerometer!
- Complimentary filter:
  - $\theta = (\theta + \theta_g * dt) (1 - \alpha) + \theta_a \alpha$

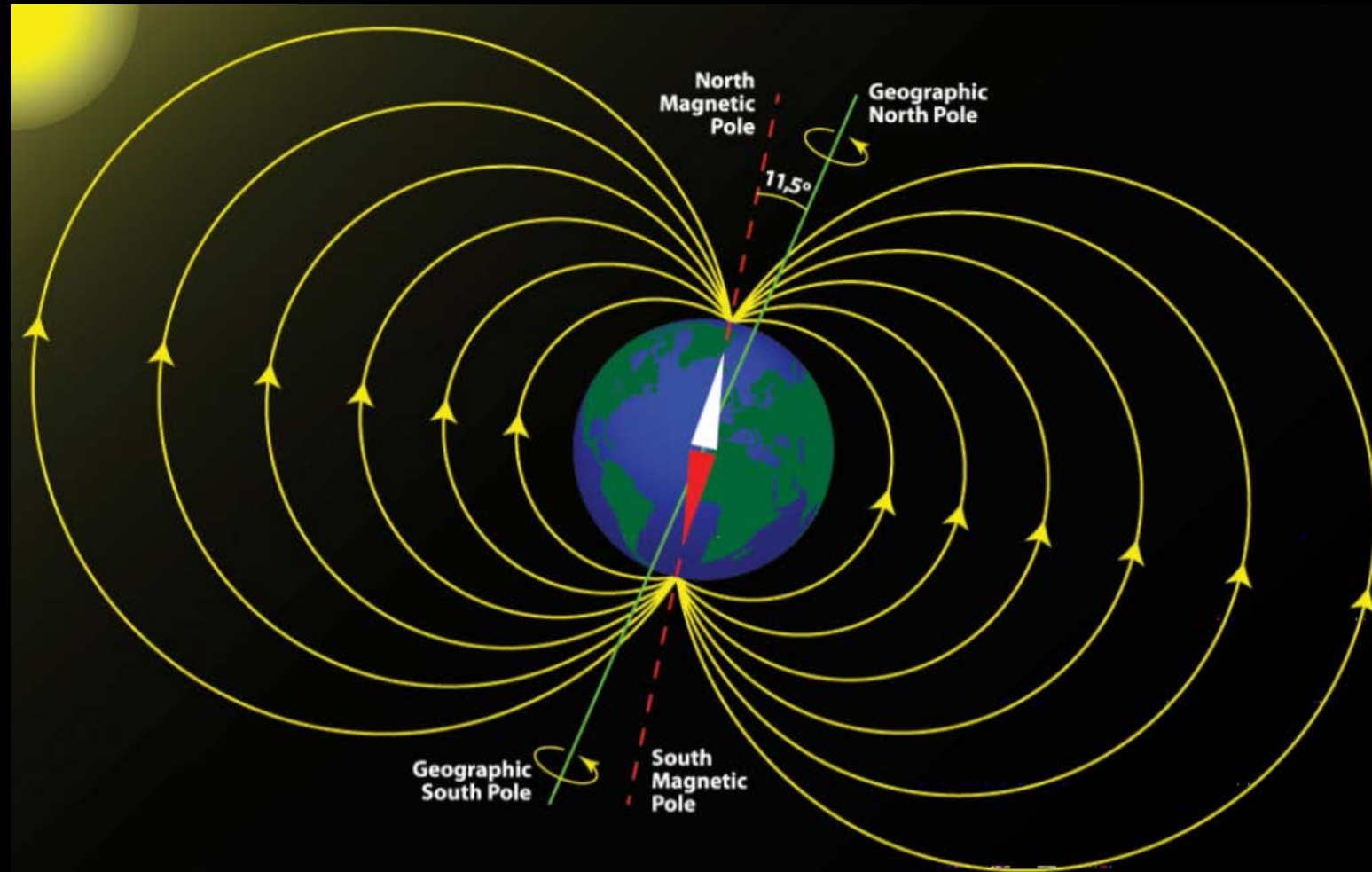


## *Can we also estimate yaw?*

- Yes! (but there is no complementary data from the accelerometer)

# Magnetometer

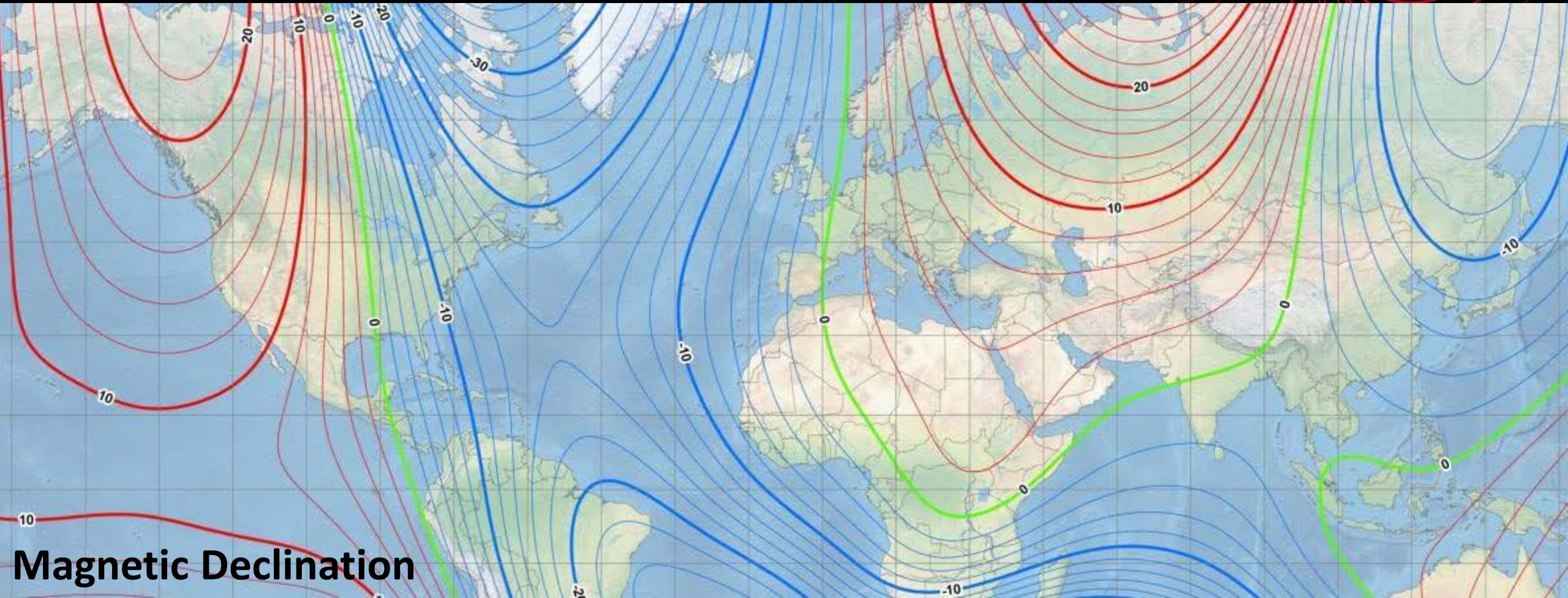
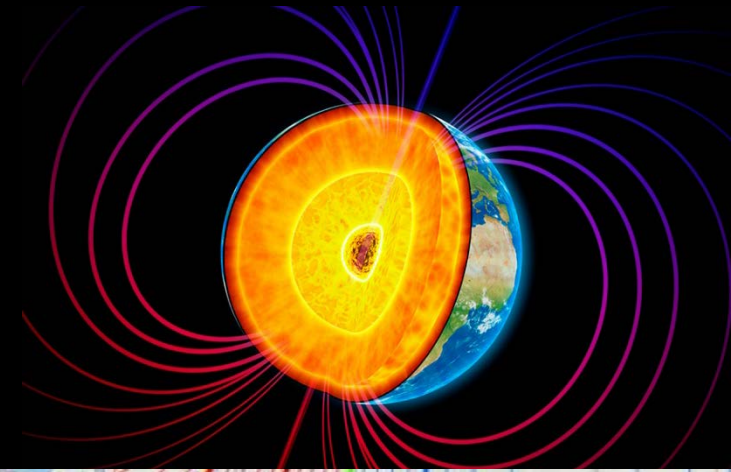
- Measure the Earth's magnetic field [Gauss] or [uT]
- The actual direction depends on latitude, longitude, and time





# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [ $\mu\text{T}$ ]
- The actual direction depends on latitude, longitude, and time
- Distortions due to metal objects or nearby EM fields



**Magnetic Declination**



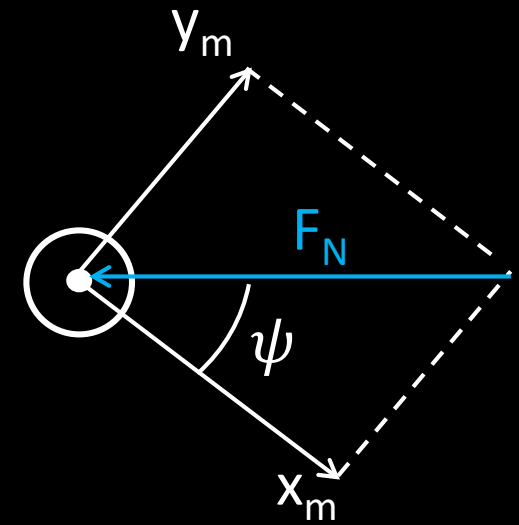
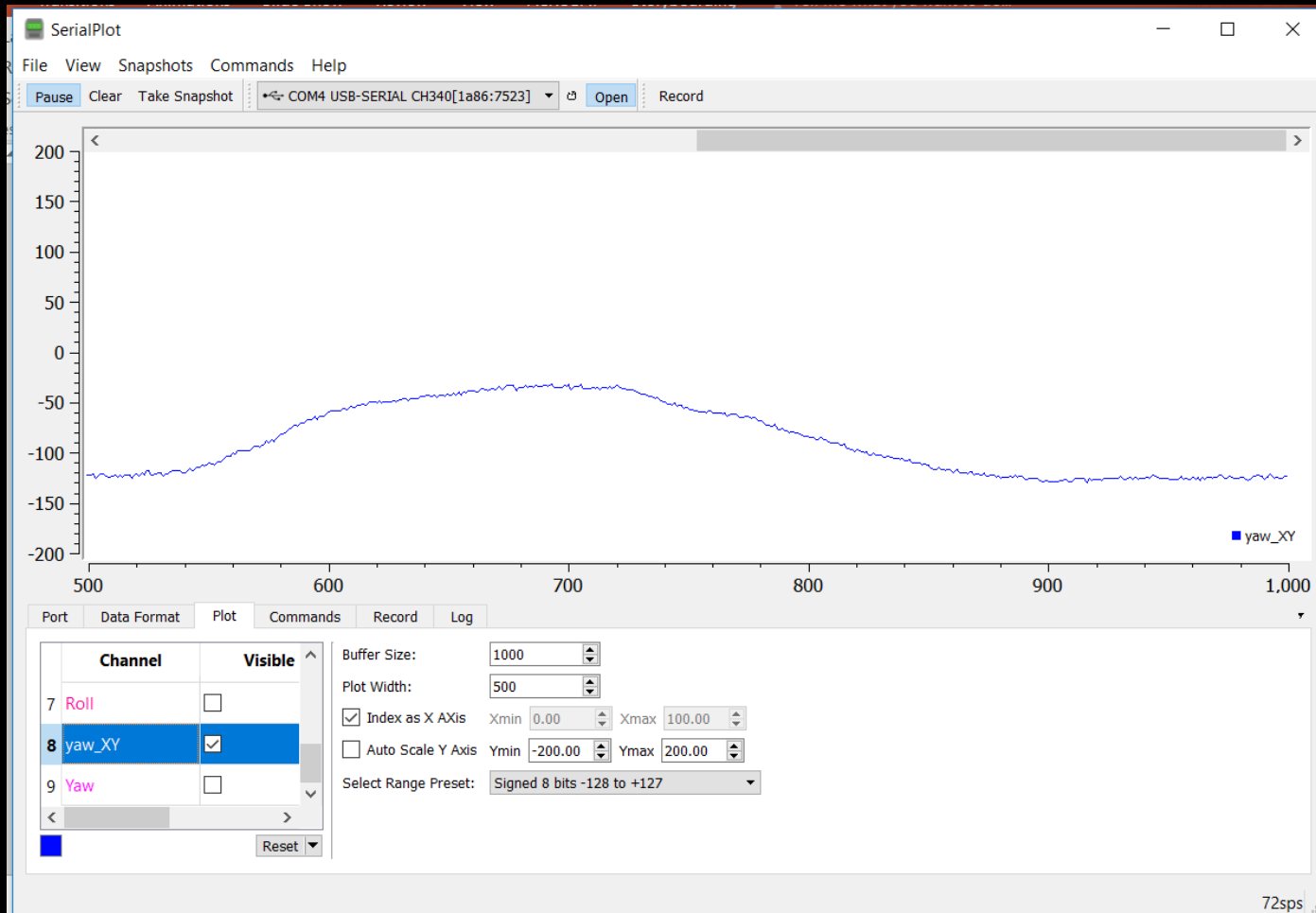
# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [ $\mu\text{T}$ ]
- The actual direction depends on latitude, longitude, and time
- Distortions due to metal objects or nearby EM fields
  - *Examples?*



# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$



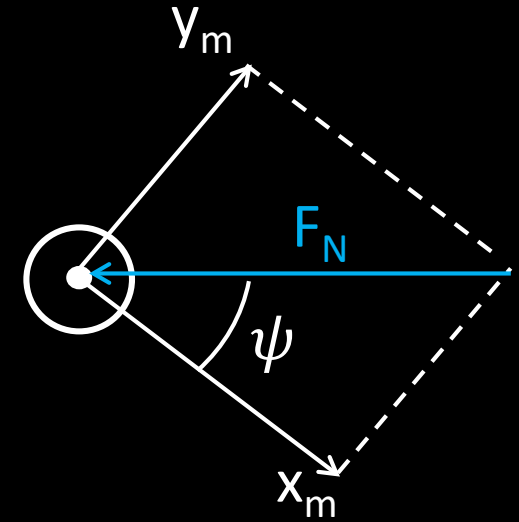
# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$
- What if you are also experiencing pitch and roll?
  - Fuse accelerometer + gyroscope + magnetometer data
- Tilt-compensated compass

$$\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = R_{x,\phi} R_{y,\theta} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

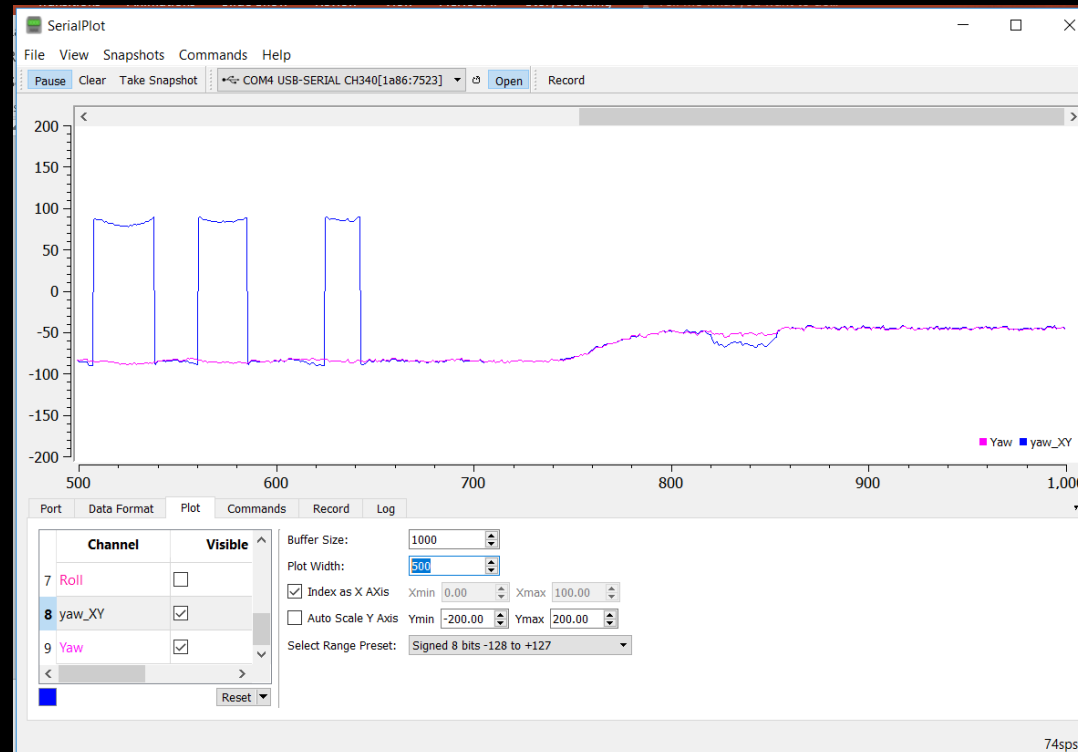
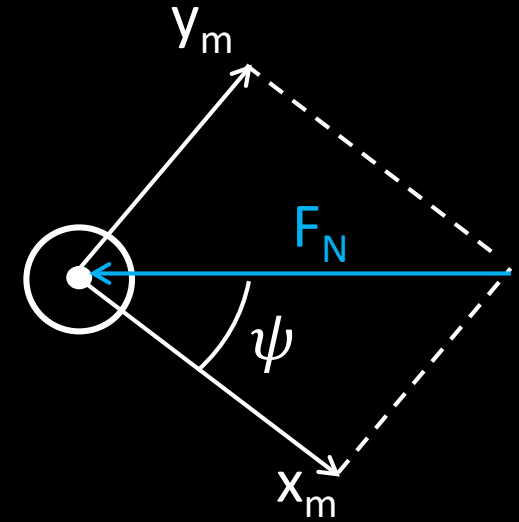
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_{x,\phi}^T R_{y,\theta}^T \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ \sin(\phi)\sin(\theta) & \cos(\phi) & \cos(\theta)\sin(\phi) \\ \cos(\phi)\sin(\theta) & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}$$

- $x = y_m * \cos(\phi) - z_m * \sin(\phi);$
- $y = x_m * \cos(\theta) + y_m * \sin(\phi) * \sin(\theta) + z_m * \cos(\phi) * \sin(\theta);$
- $\psi = \text{atan2}(y, x)$



# Magnetometer

- Measure the Earth's magnetic field [Gauss] or [uT]
- $\psi = \text{atan}\left(\frac{y_m}{x_m}\right)$
- What if you are also experiencing pitch and roll?
  - Fuse accelerometer + gyroscope + magnetometer data
- Tilt-compensated compass



# Lab 3: Characterize your Car

- Open ended...
  - But we expect *useful* data
  - Qualitative and quantitative analysis
  - Structured experiments
  - Proof and replicas
- What would be helpful to know?

Measure	<ul style="list-style-type: none"><li>• Dimension (chassis/wheels)</li><li>• Mass</li><li>• Battery life time</li></ul>
Experimental	<ul style="list-style-type: none"><li>• Static friction</li><li>• Braking distance</li><li>• Velocity range / motor supply voltage</li><li>• Acceleration range</li><li>• Tricks</li></ul>

**How would you set this up?**  
**How would you measure this?**

- Feel free to work in teams (remember to give credits)
- Set aside time every day
- Deliverable: Github page and 3-5 min presentation 9/29<sup>th</sup>

# Sources and References

- <http://www.chrobotics.com/library/accel-position-velocity>
- EE 267 Virtual Reality, by Gordon Wetzstein at Stanford University
- Analog.com
- <https://toptechboy.com/>

