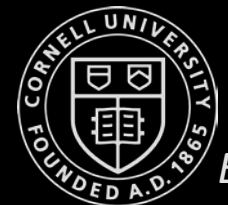
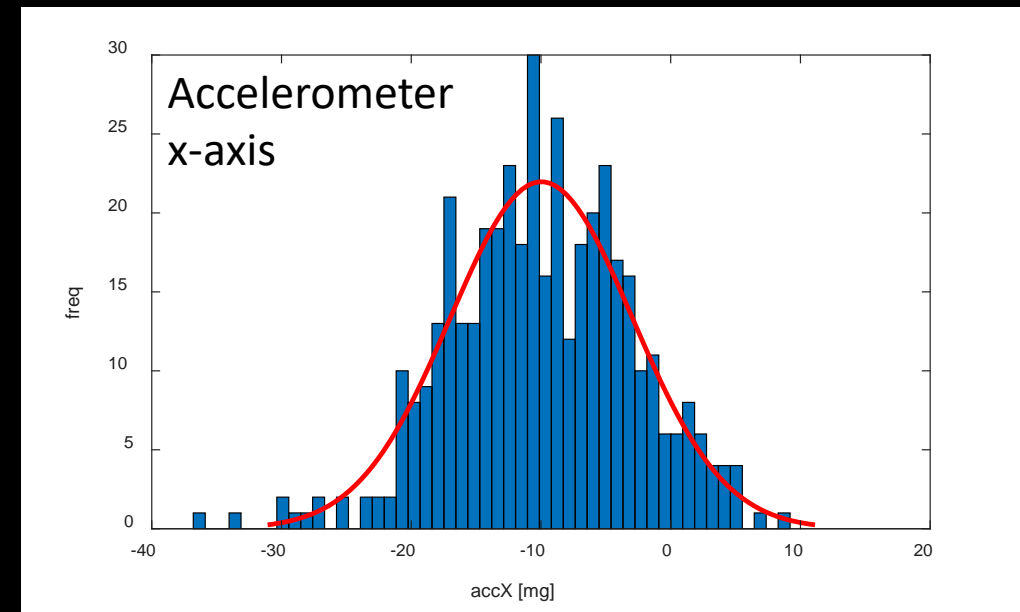
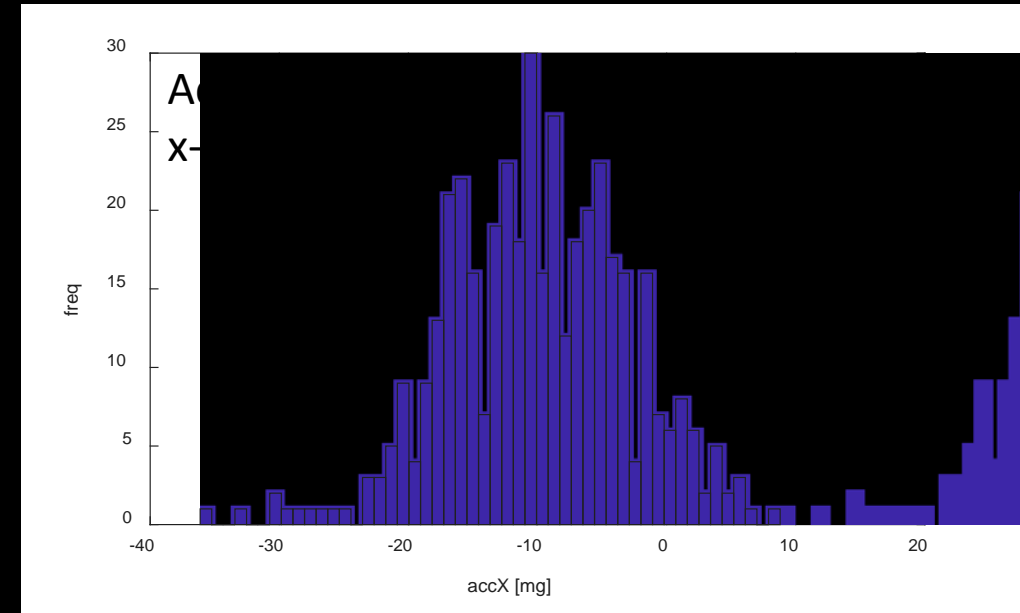


Fast Robots



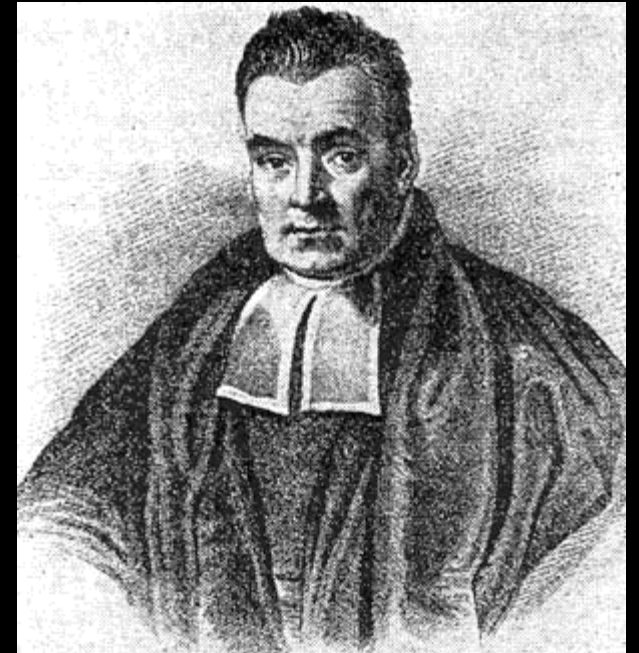
Noisy Sensors

- Example: Accelerometer
- Solution?
 - Average over multiple samples
 - mean = -9.97306mg
 - std dev = 7.0318mg
- Normal distributions
 - Described with 2 numbers
 - $[\mu \mp \sigma]$
 - Symmetric
 - Unimodal
 - Sums to unity
- Probabilistic robotics
 - Measurements are uncertain
 - Actions are uncertain
 - States are uncertain

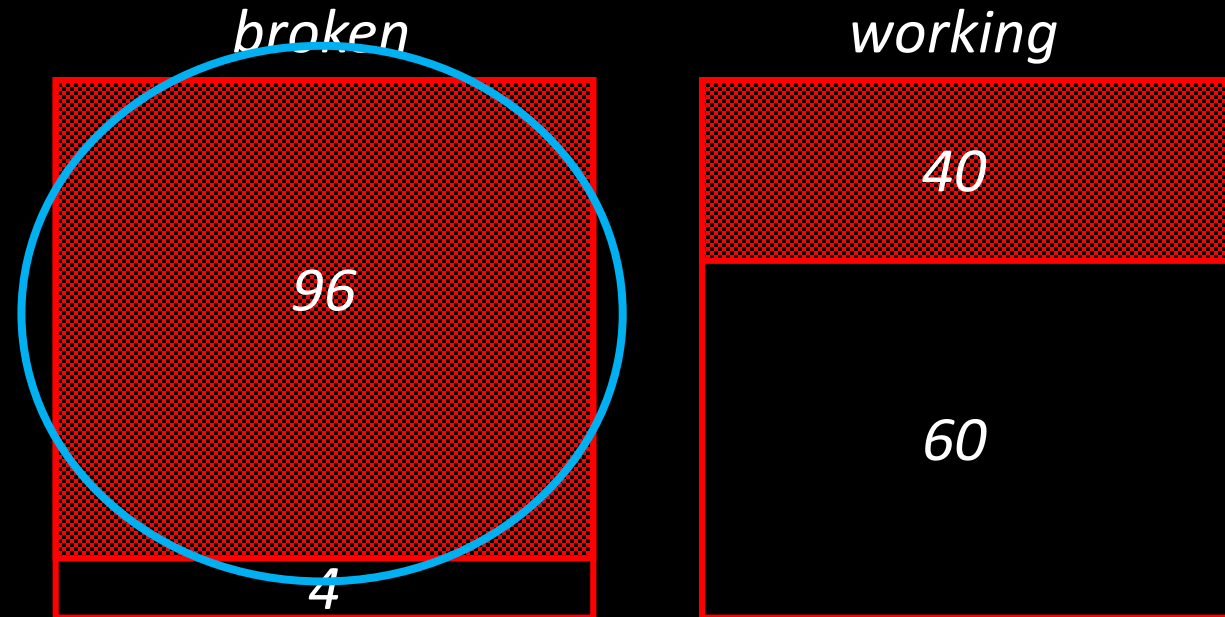


Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

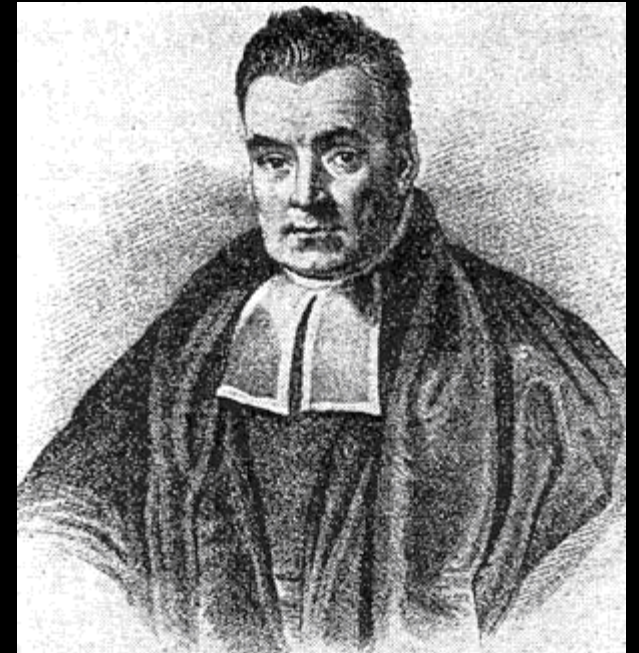



- no motion
- motion

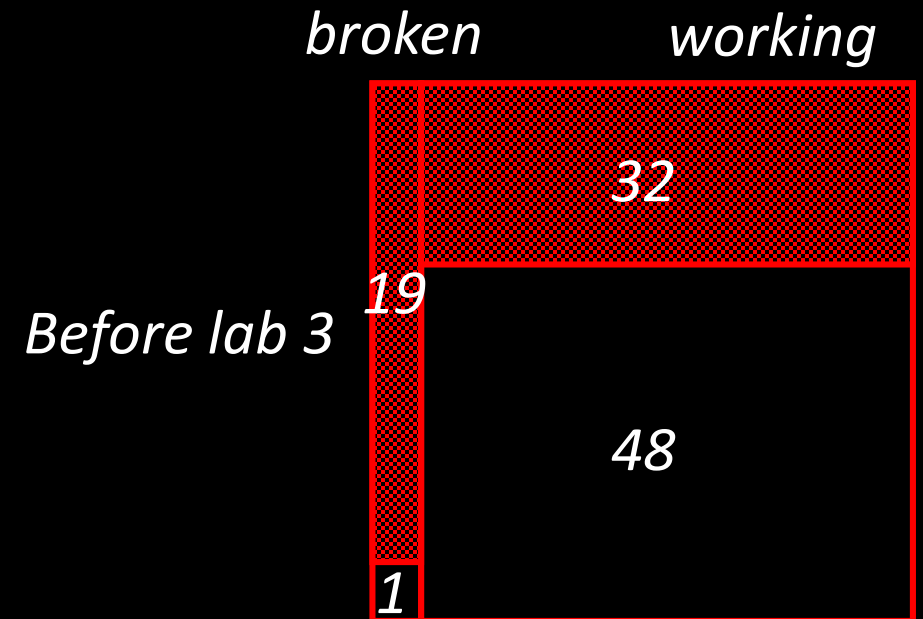
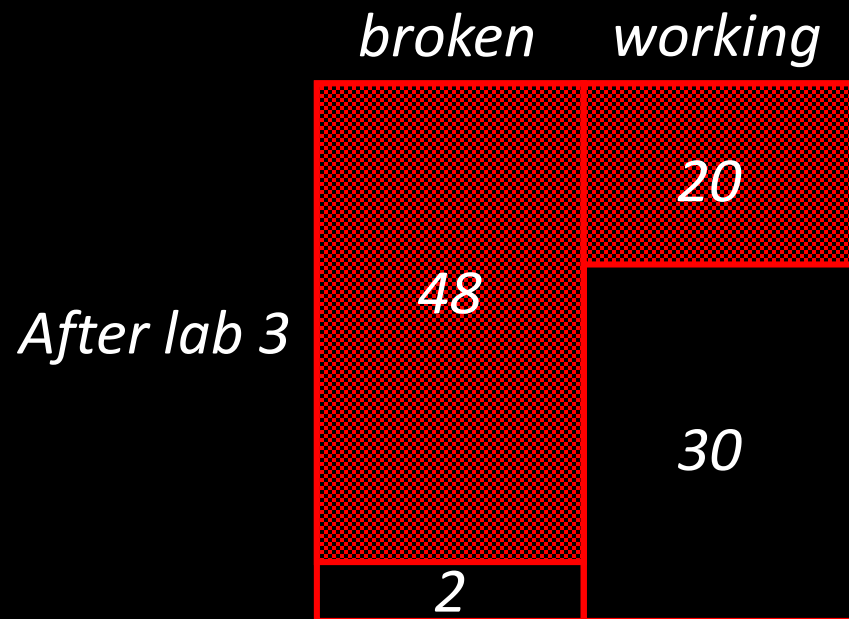


Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?



 no motion
 motion



Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?
- **Translate to math**
 - $P(\text{something}) = \frac{\#\text{something}}{\#\text{everything}}$
 - Before lab 3:
 - $P(\text{broken}) = \frac{\#\text{broken}}{\#\text{kits}} = \frac{20}{100} = 0.2$
 - $P(\text{working}) = \frac{\#\text{working}}{\#\text{kits}} = \frac{80}{100} = 0.8$
 - After lab 3:
 - $P(\text{broken}) = \frac{\#\text{broken}}{\#\text{kits}} = \frac{50}{100} = 0.5$
 - $P(\text{working}) = \frac{\#\text{working}}{\#\text{kits}} = \frac{50}{100} = 0.5$

	<i>broken</i>	<i>working</i>
	48	20
	2	30

After lab 3

	<i>broken</i>	<i>working</i>
	19	32
	1	48

Before lab 3

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?
- **Conditional Probability**
 - If you know that the robot is broken, what is the probability that it stopped moving?
 - $P(\text{no motion} \mid \text{broken}) = \frac{\#\text{broken and no motion}}{\#\text{broken}}$
 - After lab 3 $= \frac{48}{50} = 0.96$
 - $P(\text{no motion} \mid \text{working}) = \frac{\#\text{working and no motion}}{\#\text{working}}$
 - After lab 3 $= \frac{20}{50} = 0.40$

	<i>broken</i>	<i>working</i>
	48	20
	2	30

After lab 3

	<i>broken</i>	<i>working</i>
	19	32
	1	48

Before lab 3

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?
- **Conditional Probability**
 - If you know that the robot is broken, what is the probability that it stopped moving?
 - $P(\text{no motion} \mid \text{broken}) = \frac{\#\text{broken and no motion}}{\#\text{broken}}$
 - Before lab 3 $= \frac{19}{20} = 0.96$
 - $P(\text{no motion} \mid \text{working}) = \frac{\#\text{working and no motion}}{\#\text{working}}$
 - Before lab 3 $= \frac{32}{80} = 0.40$

	<i>broken</i>	<i>working</i>
	48	20
	2	30

After lab 3

	<i>broken</i>	<i>working</i>
	19	32
	1	48

Before lab 3

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

- **Conditional Probability**

- If you know that the robot is broken, what is the probability that it stopped moving?
- $P(A|B)$ is the probability of A, given B
- Note: $P(A|B)$ is not equal to $P(B|A)$
 - $P(\text{cute}|\text{puppy}) \neq P(\text{puppy}|\text{cute})$

	<i>broken</i>	<i>working</i>
<i>After lab 3</i>	48	20
	2	30

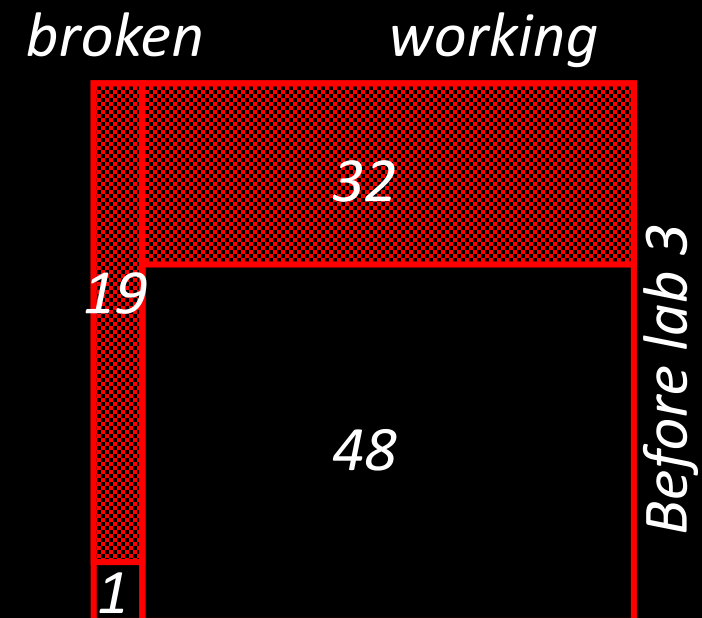
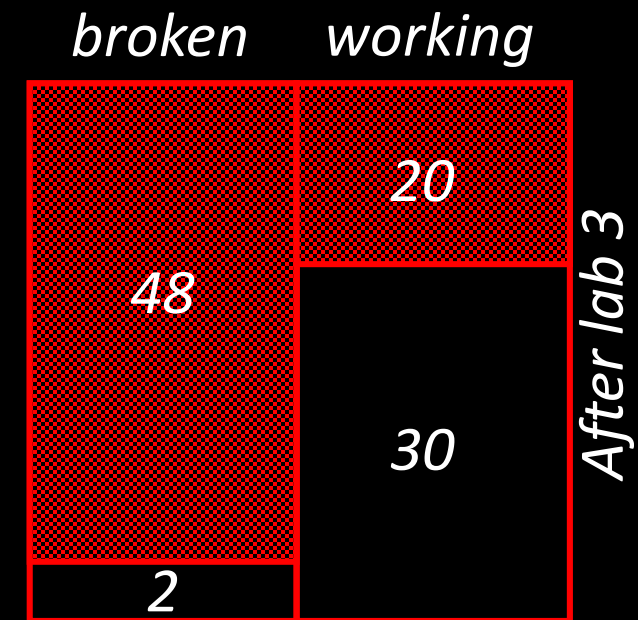
	<i>broken</i>	<i>working</i>
<i>Before lab 3</i>	19	32
	1	48

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

- **Joint Probability**

- What is the probability that the robot is both broken and not moving?
- $P(\text{broken and not moving})$
= $P(\text{broken}) * P(\text{not moving} \mid \text{broken})$
= $0.5 * 0.96 = 0.48$



Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

	<i>broken</i>	<i>working</i>
<i>After lab 3</i>	48	20
	2	30

Joint Probability

- What is the probability that the robot is both broken and not moving?

- $P(\text{broken and not moving})$
 $= P(\text{broken}) * P(\text{not moving} \mid \text{broken})$
 $= 0.20 * 0.96 = 0.192$

- $P(\text{working and moving})$
 $= P(\text{working}) * P(\text{moving} \mid \text{working})$
 $= 0.80 * 0.60 = 0.48$

$P(\text{working}) = 0.80$

	<i>broken</i>	<i>working</i>
<i>Before lab 3</i>	19	32
	1	48

$P(\text{broken}) = 0.20$

$P(\text{broken and not moving}) = 0.19$

$P(\text{working and moving}) = 0.48$

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

	<i>broken</i>	<i>working</i>
<i>After lab 3</i>	48	20
	2	30

- **Joint Probability**

- What is the probability that the robot is both broken and not moving?

- $P(A, B) = P(A \cap B) = P(A \text{ and } B)$
- $P(A \cap B) = P(A) * P(B | A)$
- $P(A \cap B) = P(B \cap A)$

	<i>broken</i>	<i>working</i>
<i>Before lab 3</i>	19	32
	1	48

$P(\text{broken}) = 0.20$
 $P(\text{working}) = 0.80$
 $P(\text{broken and not moving}) = 0.19$
 $P(\text{working and moving}) = 0.48$

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?

• Marginal Probability

- $P(\text{moving})$
 $= P(\text{broken and moving}) + P(\text{working and moving})$
 $= 1/100 + 48/100 = 0.49$
- $P(\text{not moving})$
 $= 19/100 + 32/100 = 0.51$

	<i>broken</i>	<i>working</i>	
	48	20	<i>After lab 3</i>
	2	30	

$P(\text{working}) = 0.80$

	<i>broken</i>	<i>working</i>	
	19	32	<i>Before lab 3</i>
	1	48	

$P(\text{broken}) = 0.20$

$P(\text{broken and not moving}) = 0.19$

$P(\text{working and moving}) = 0.48$

Bayesian Inference

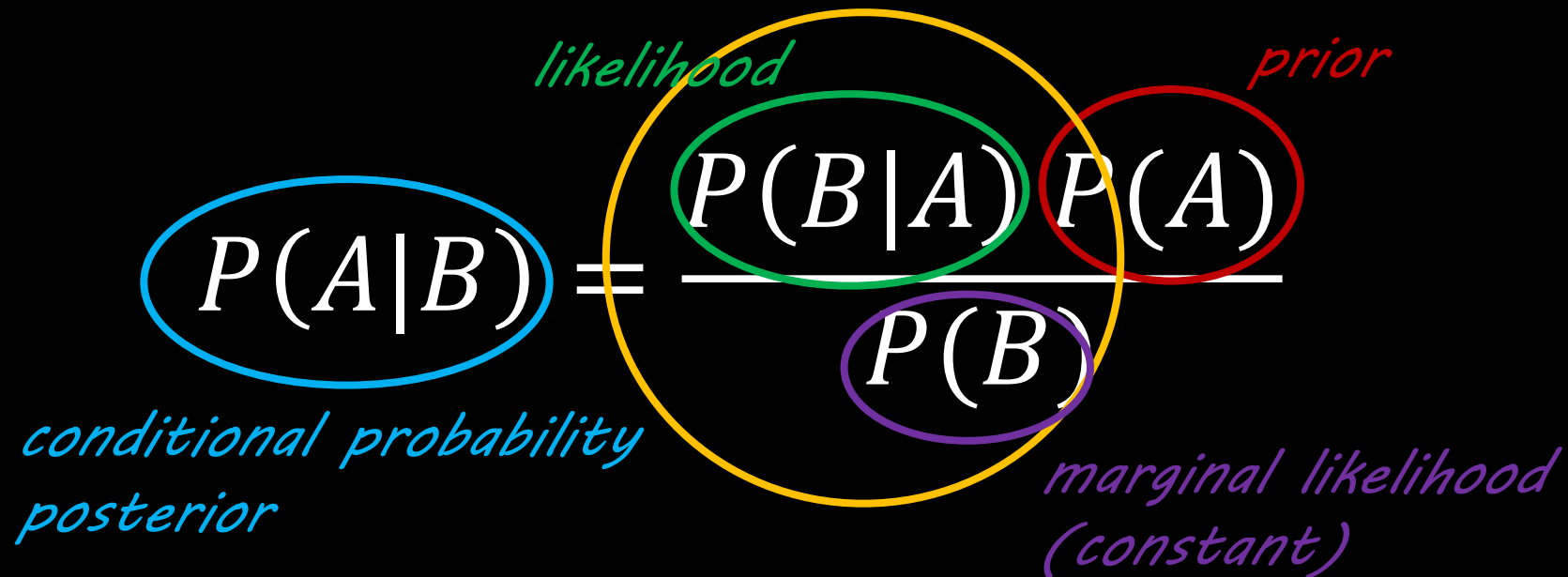
- Bayesian inference = guessing in the style of Bayes
- Example
 - *Campuswire*: The robot stopped moving, the hardware is broken, send me new parts
 - What is the probability that the robot is broken, given that it stopped moving?
 - $P(\text{broken} \mid \text{not moving}) = ???$
- $P(\text{broken and not moving})$
 $= P(\text{not moving}) * P(\text{broken} \mid \text{not moving})$
- $P(\text{not moving and broken})$
 $= P(\text{broken}) * P(\text{not moving} \mid \text{broken})$
- $P(\text{broken} \mid \text{not moving}) = \frac{P(\text{broken}) * P(\text{not moving} \mid \text{broken})}{P(\text{not moving})}$
- Before lab 3 $= 0.2 * 0.96 / 0.51 = 0.38$
- After lab 3 $= 0.5 * 0.96 / 0.68 = 0.71$

	<i>broken</i>	<i>working</i>	
	48	20	<i>After lab 3</i>
	2	30	

	<i>broken</i>	<i>working</i>	
	19	32	<i>Before lab 3</i>
	1	48	

Bayesian Inference

- Bayesian inference = guessing in the style of Bayes



The diagram illustrates the Bayesian inference formula with color-coded annotations. The formula is $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$. The terms are annotated as follows: $P(A|B)$ is labeled as "conditional probability posterior" (blue); $P(B|A)$ is labeled as "likelihood" (green); $P(A)$ is labeled as "prior" (red); and $P(B)$ is labeled as "marginal likelihood (constant)" (purple). A yellow circle encompasses the likelihood and prior terms, and a blue circle encompasses the posterior term.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

conditional probability posterior

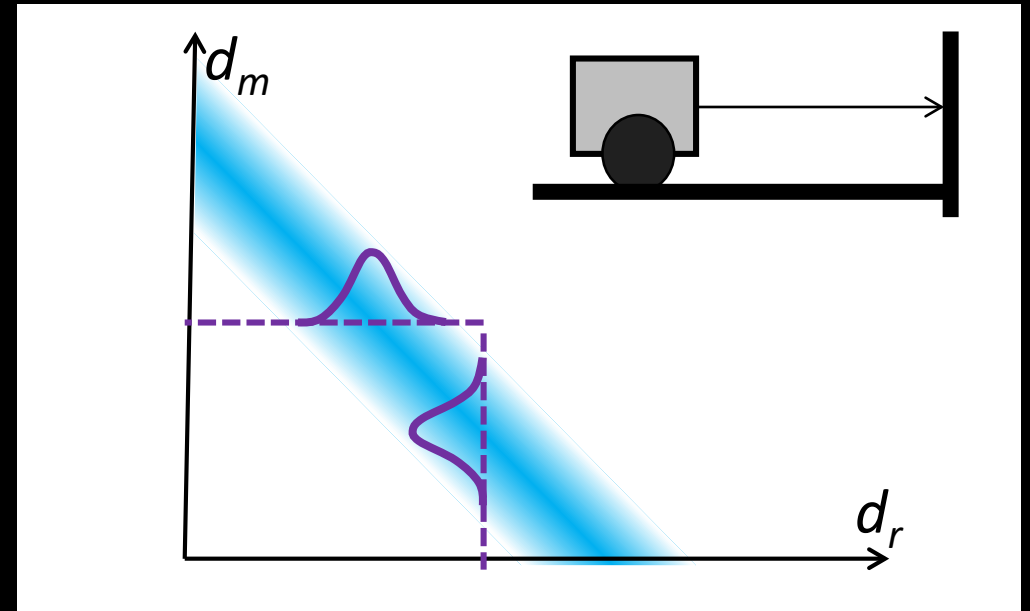
likelihood

prior

marginal likelihood (constant)

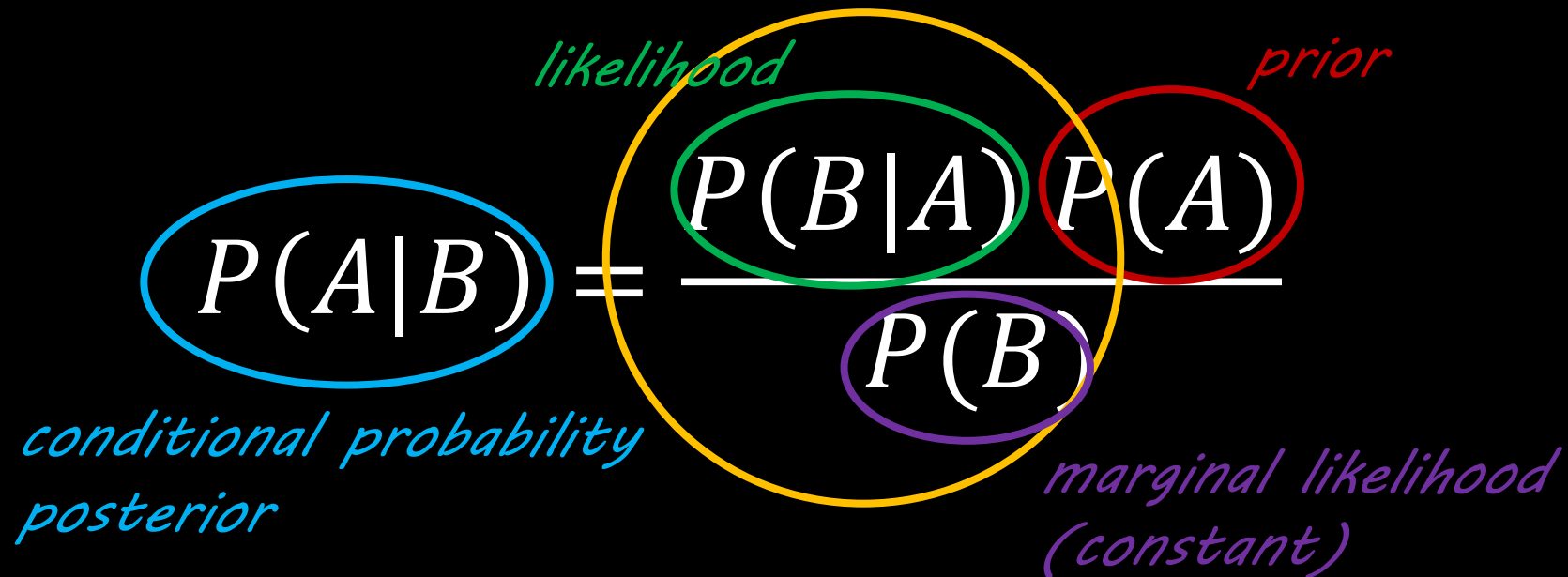
Exercise

- **Conditional probability**
 - You meet a guy, and he says he has a sibling, what is the probability that the sibling is female?
 - guy/girl
 - guy/guy
 - girl/guy
 - girl/girl (<ruled out)
 - 33%
- Independent / dependent variables



Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard

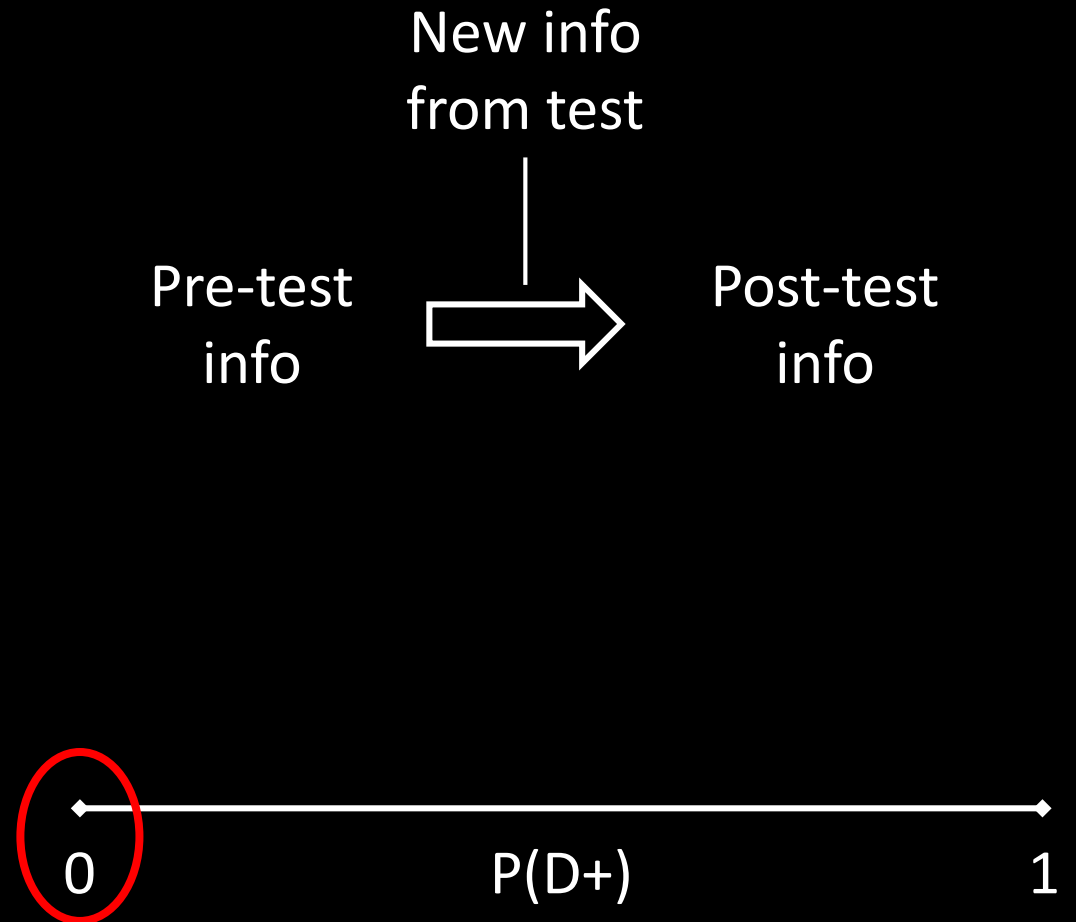


The diagram illustrates the Bayesian Inference formula: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$. The components are annotated with colored circles and labels:

- $P(A|B)$ is circled in blue and labeled "conditional probability posterior".
- $P(B|A)$ is circled in green and labeled "likelihood".
- $P(A)$ is circled in red and labeled "prior".
- $P(B)$ is circled in purple and labeled "marginal likelihood (constant)".
- A yellow circle encompasses the entire fraction $\frac{P(B|A)P(A)}{P(B)}$.

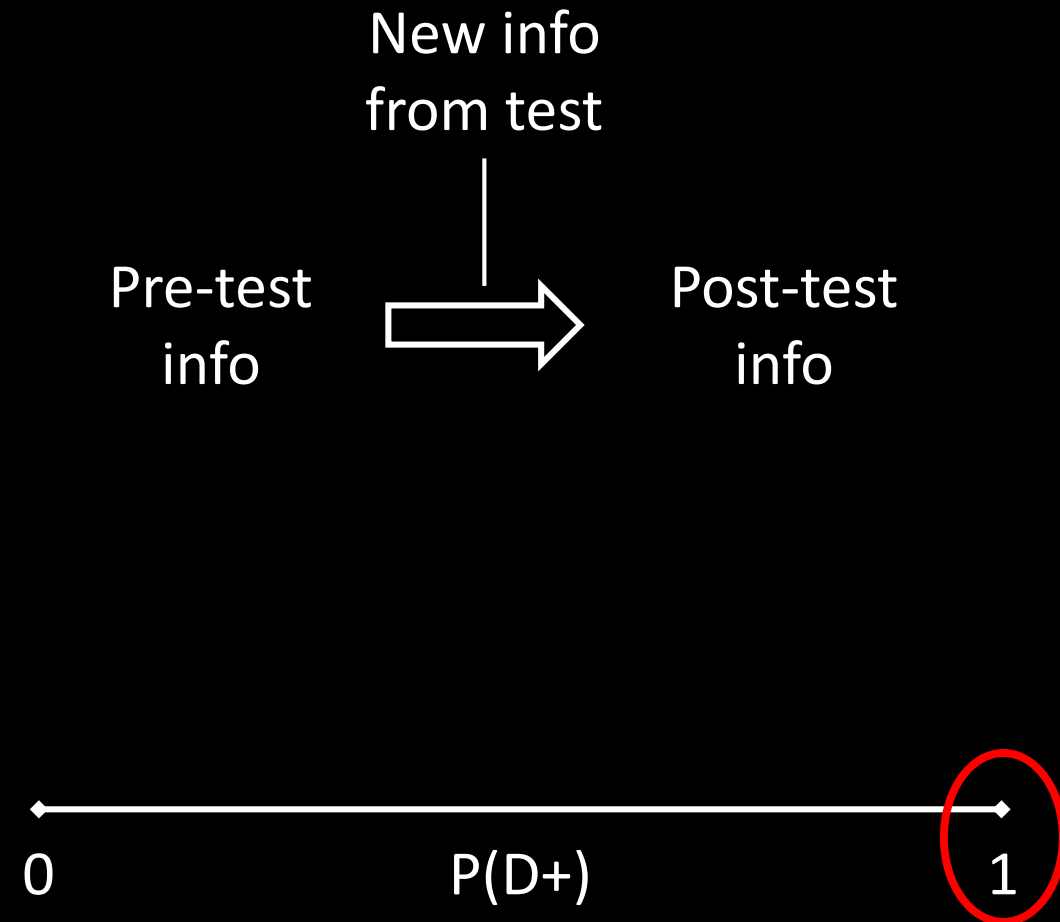
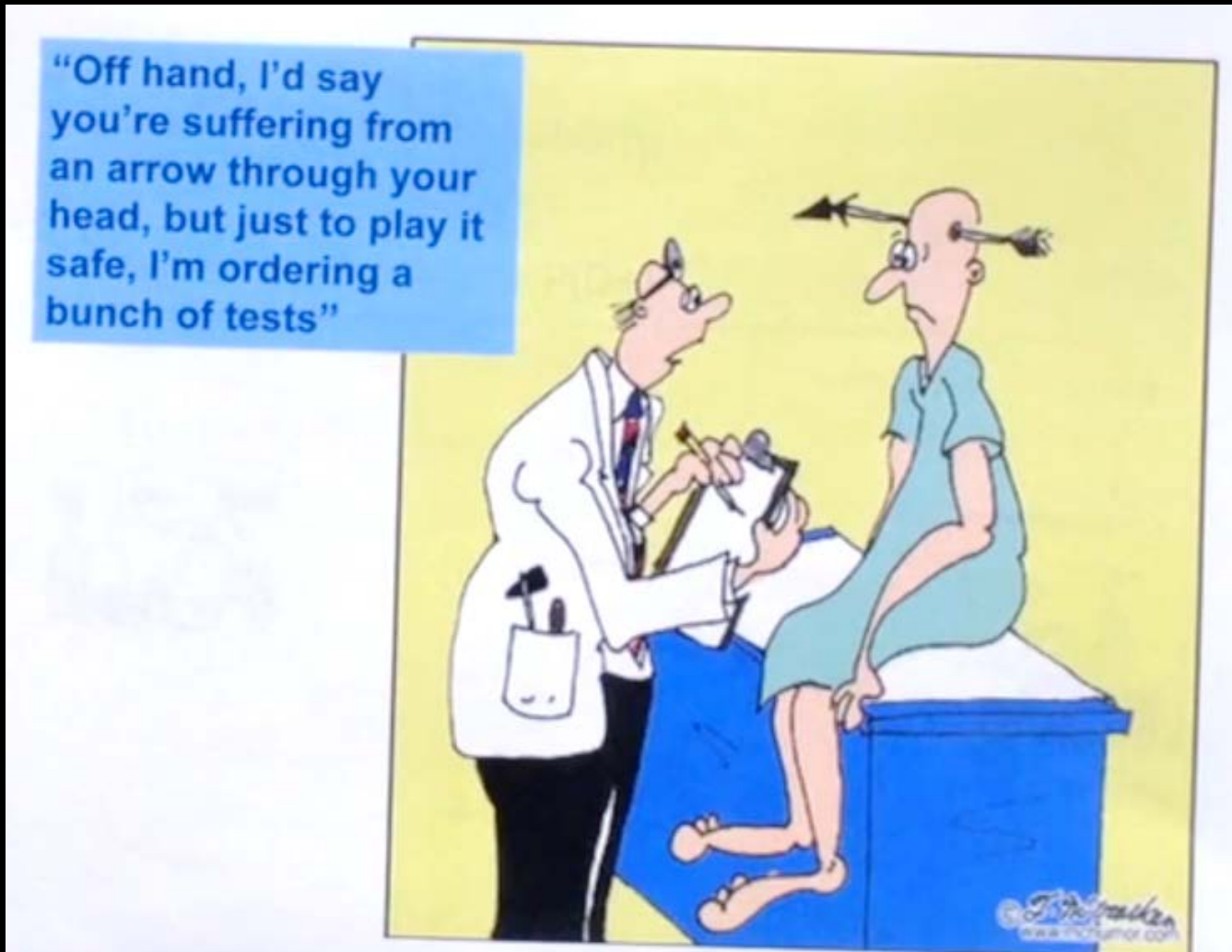
Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard



Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard



Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard



- 100 people go to the doctor with a tick bite, worried they have Lyme disease
- A diagnostic test reveals 16 positives, 84 negative
- Early tests are inaccurate:
 - 40% of sick people will test positive
 - 10% of healthy people will test positive

Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard

Observation	Underlying truth	
	D+	D-
T+	16	
T-	84	80

- 100 people go to the doctor with a tick bite, worried they have Lyme disease
- A diagnostic test reveals 16 positives, 84 negative
- Early tests are inaccurate:
 - 40% of sick people will test positive
 - 10% of healthy people will test positive
- Underlying truth:
 - 20% are sick
 - 80% are healthy

Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard

Observation

Underlying truth

D+

D-

T+

16

8

8

$$= \frac{8}{16} = 50\% = P(D + | T +)$$

T-

84

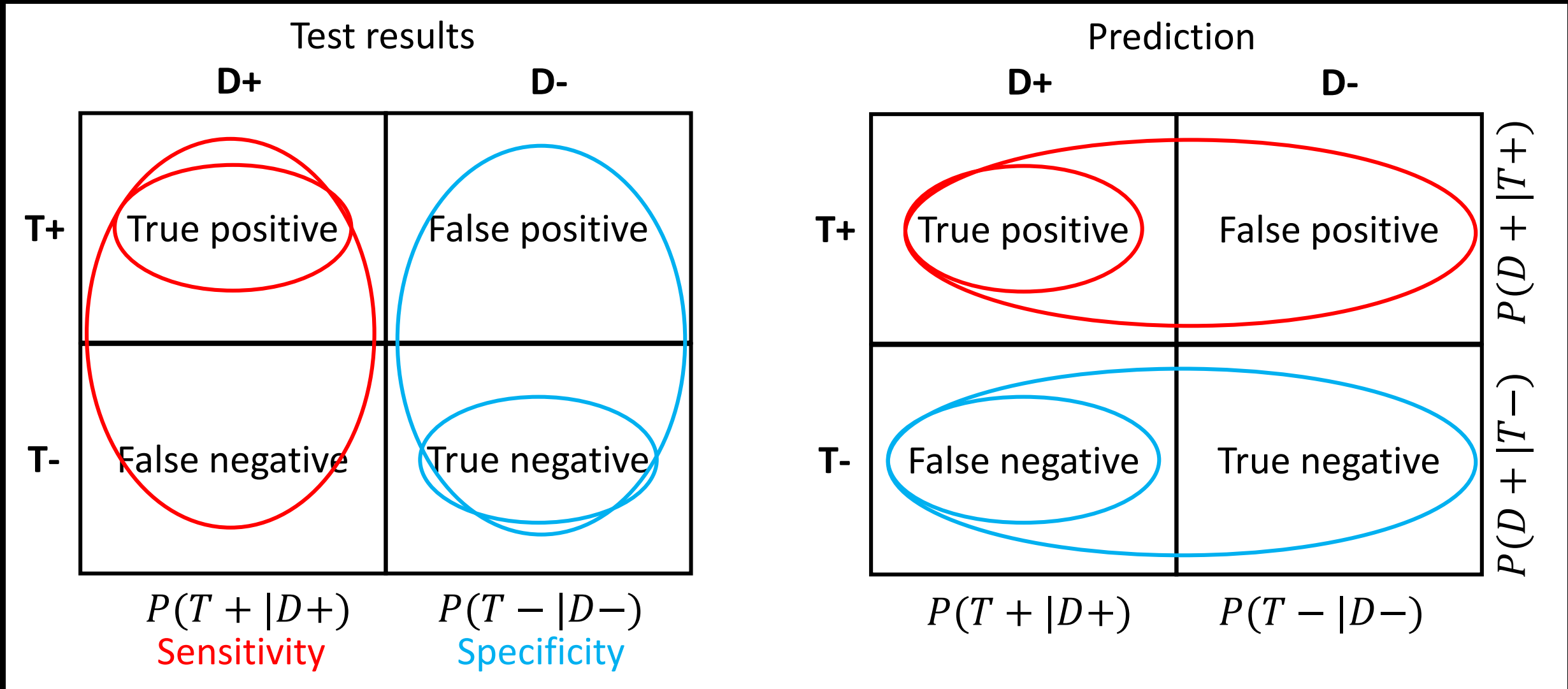
12

72

$$= \frac{12}{84} = 14\% = P(D + | T -)$$

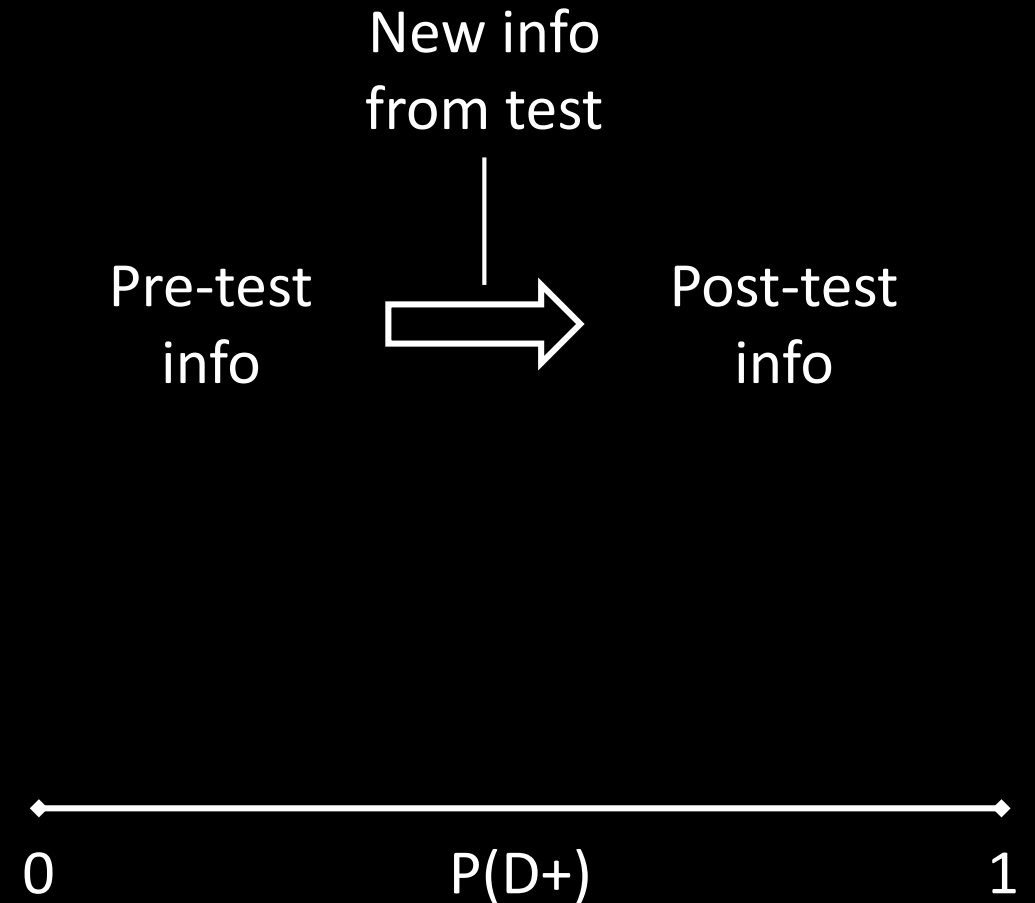
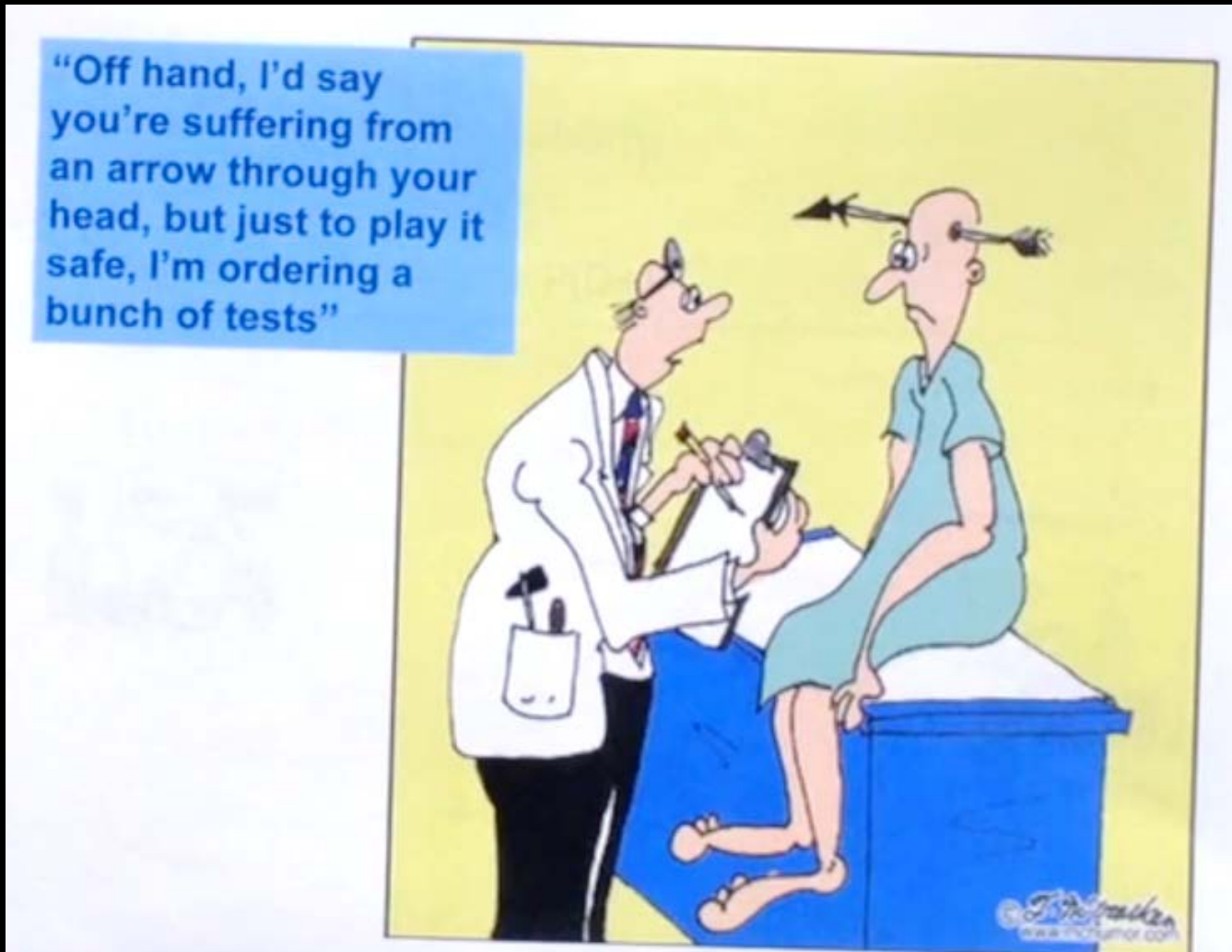
Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard



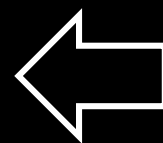
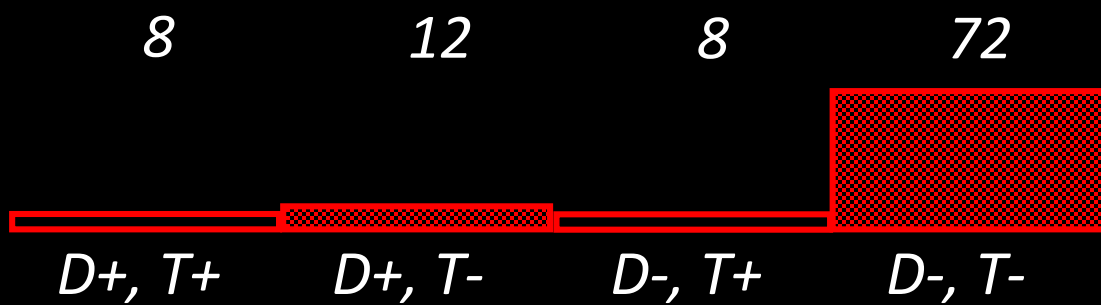
Bayesian Inference

- Unrelated example, borrowed from “Bayes with Beans” by Myriam Hunink, Harvard



Probability Distribution

- Beliefs

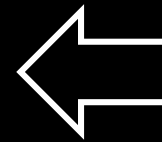
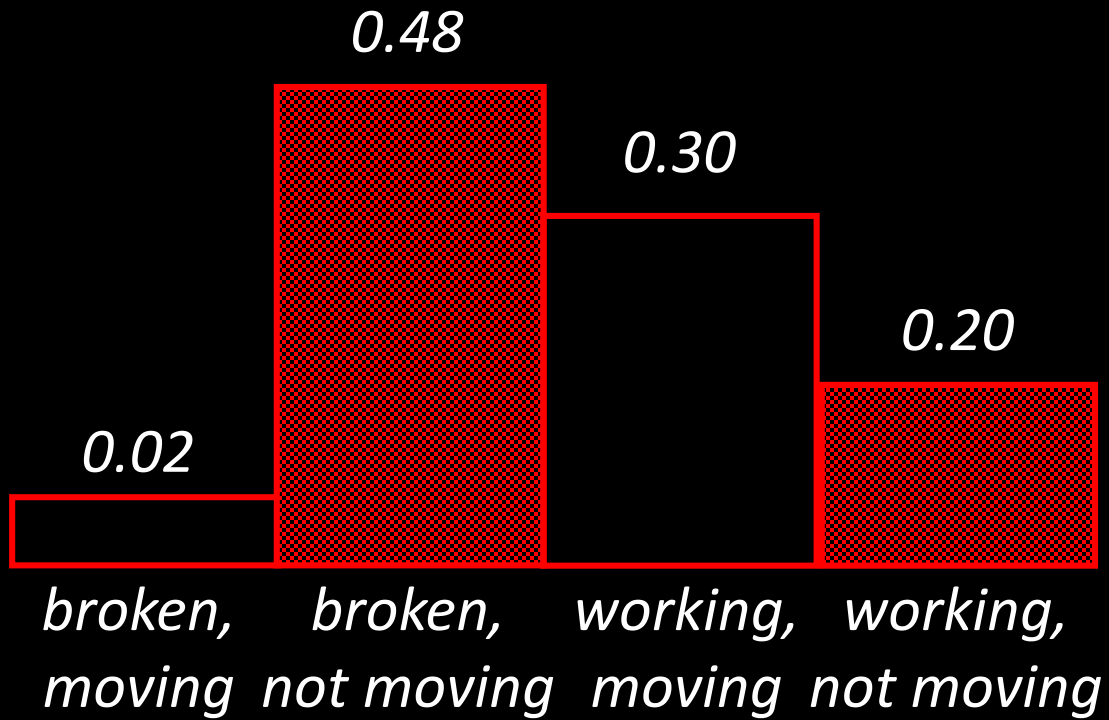


	D+	D-
T+	8	8
T-	12	72

A 2x2 contingency table with a red border. The top row is labeled T+ and the bottom row is labeled T-. The left column is labeled D+ and the right column is labeled D-. The cells contain the values 8, 8, 12, and 72. The bottom-right cell (T-, D-) is filled with a red hatched pattern.

Probability Distribution

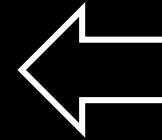
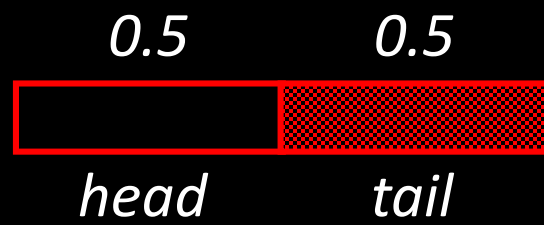
- Beliefs



	<i>broken</i>	<i>working</i>
<i>After lab 3</i>	48	20
	2	30

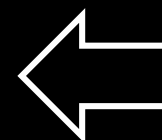
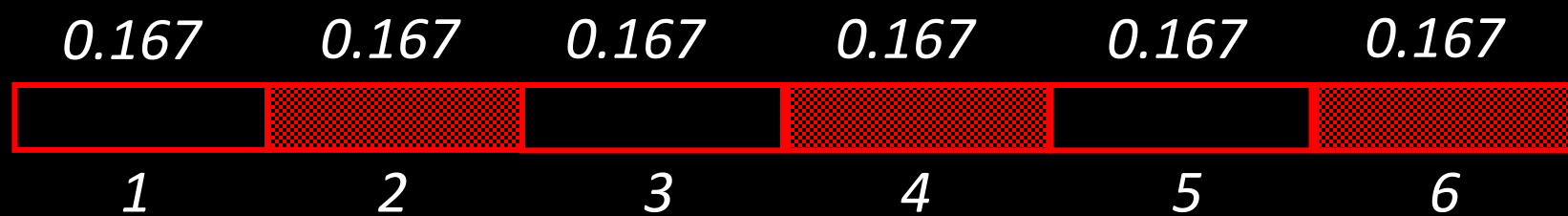
Probability Distribution

- Beliefs



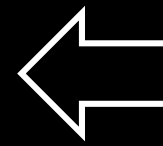
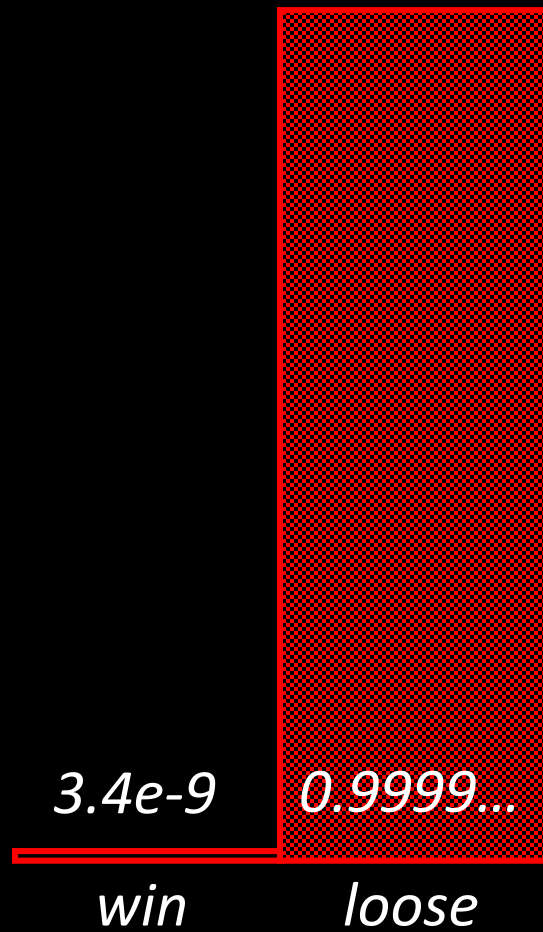
Probability Distribution

- Beliefs



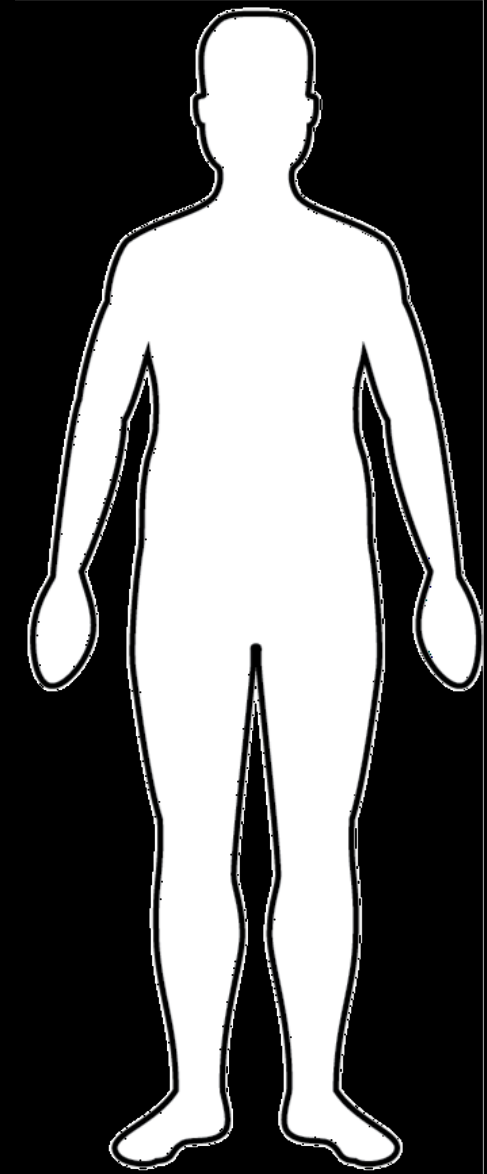
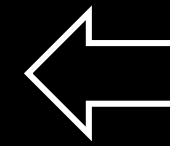
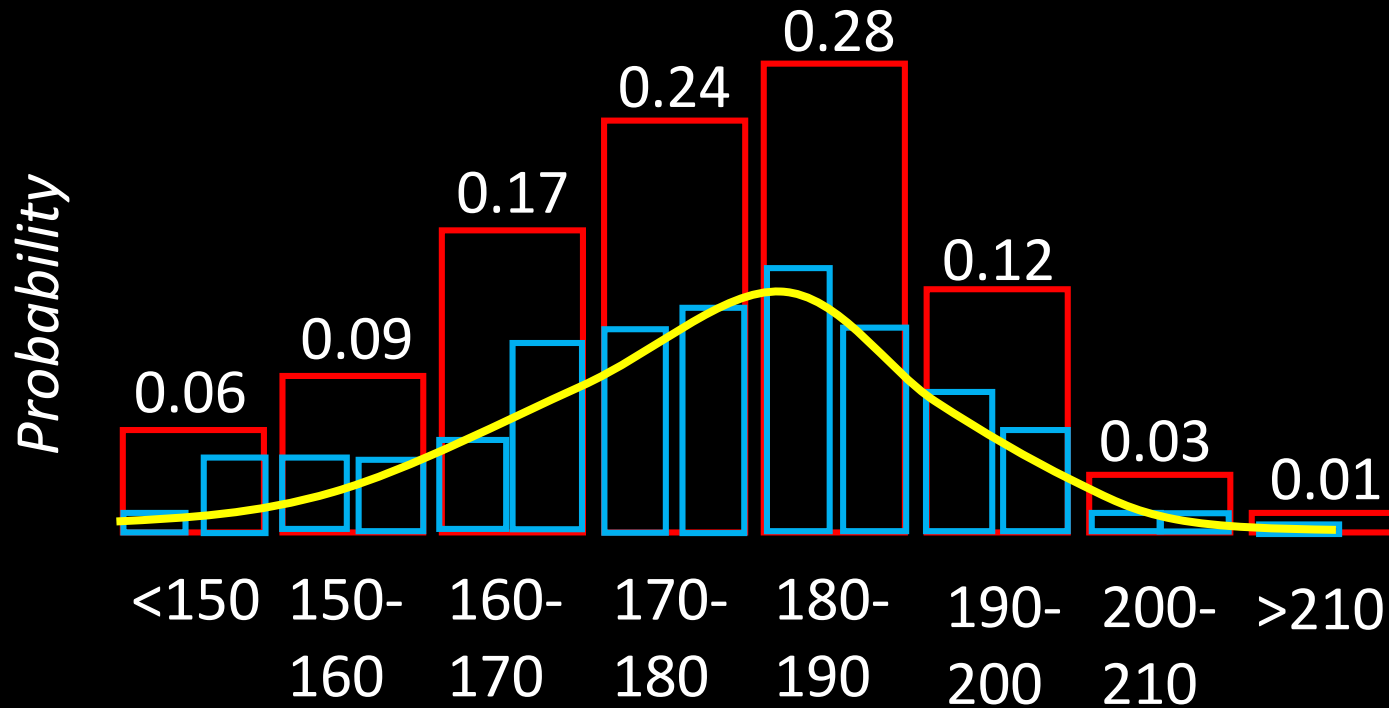
Probability Distribution

- Beliefs



Probability Distribution

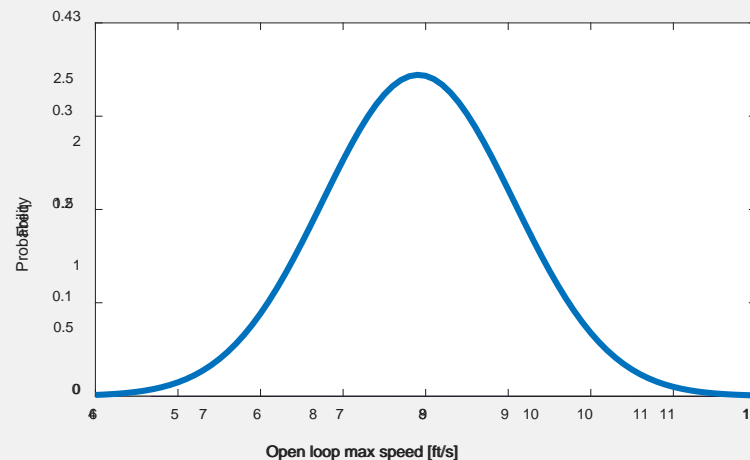
- Beliefs
- Discrete -> continuous *probability distribution*
 - Mean, median, most common value, etc.



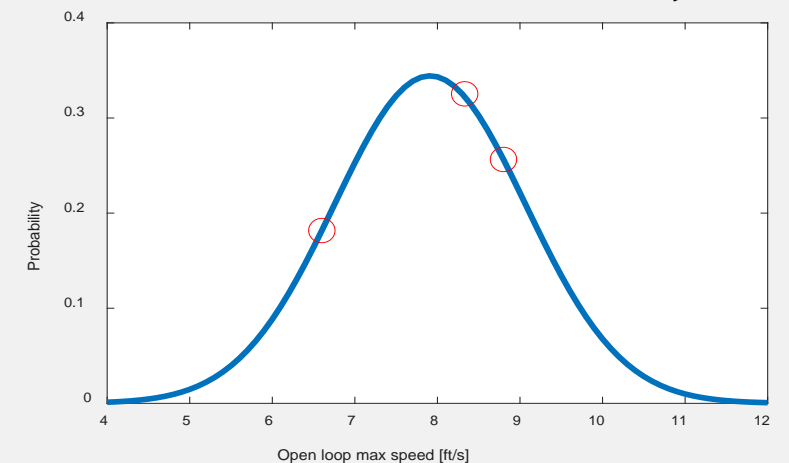
Probability Distributions

- What is the maximum speed of your robot?
 - You weigh 8.8 ft/s, 6.6 ft/s, 8.33 ft/s, but what is the actual value?
- Frequentist Statistics
 - Mean: $\mu = (8.8+6.6+8.33)/3 = 7.91$
 - Variance: $\sigma^2 = ((8.8-7.91)^2 + (6.6-7.91)^2 + (8.33-7.91)^2)/(3-1) = 1.35$
 - Standard deviation: $\sigma = \text{sqrt}(\sigma^2) = 1.16$
 - Standard error: $\sigma / \text{sqrt}(3) = 0.67$
- Bayesian Statistics
 - Probably 7.91ft/s...

Values from lab 3



What you observe (7.91 ± 1.16 ft/s)



Probability Distributions

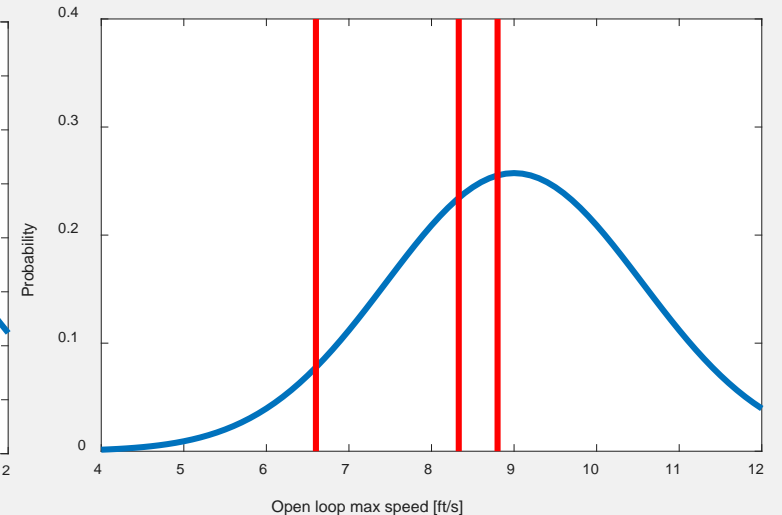
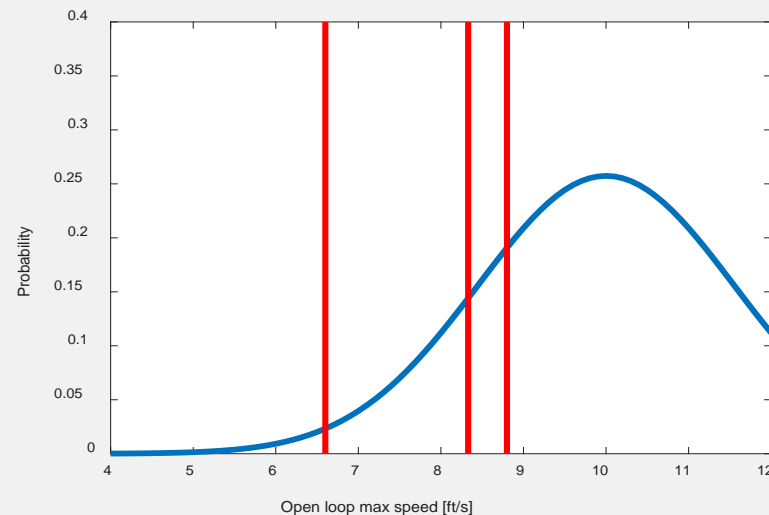
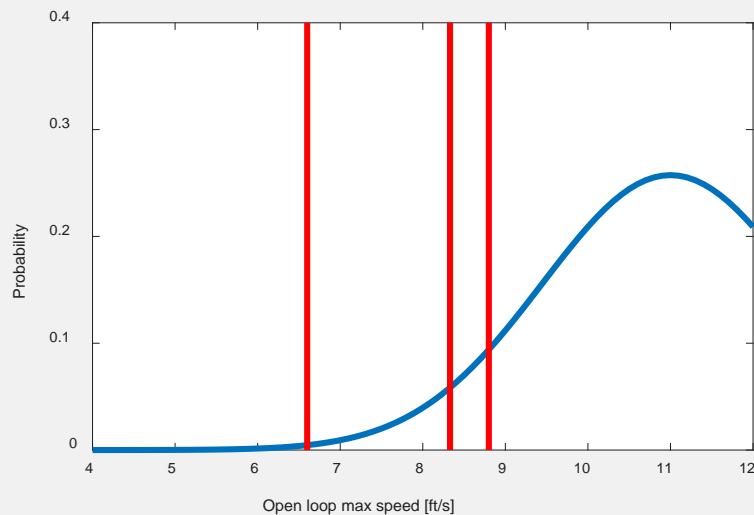
- Use Bayes theorem
- Instead of A and B
 - Substitute “s” for the actual speed
 - Substitute “m” for the measurements
- $P(s)$ is our prior
- $P(m | s)$ is the likelihood associated with those measurements
- $P(s | m)$ is what we believe about the speed given those measurements
- $P(m)$ is the marginal likelihood
- Procedure:
 - Start with a belief
 - Update it
 - End up with a new belief!

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

Probability Distributions

- Use Bayes theorem
- Start by assuming nothing
 - $P(s) = \text{uniform}$
 - $P(s | m) = P(m | s) * c_1 / c_2$
 - Simplified: $P(s | m) = P(m | s)$
 - *Guess!* What if the actual max speed is 11 ft/s?
 - $P(s=11 | m=[6.6, 8.33, 8.8]) = P(m=[6.6, 8.33, 8.8] | s=11)$
 - $P(m = 6.6 | s = 11) * P(m = 8.33 | s = 11) * P(m = 8.8 | s = 11)$

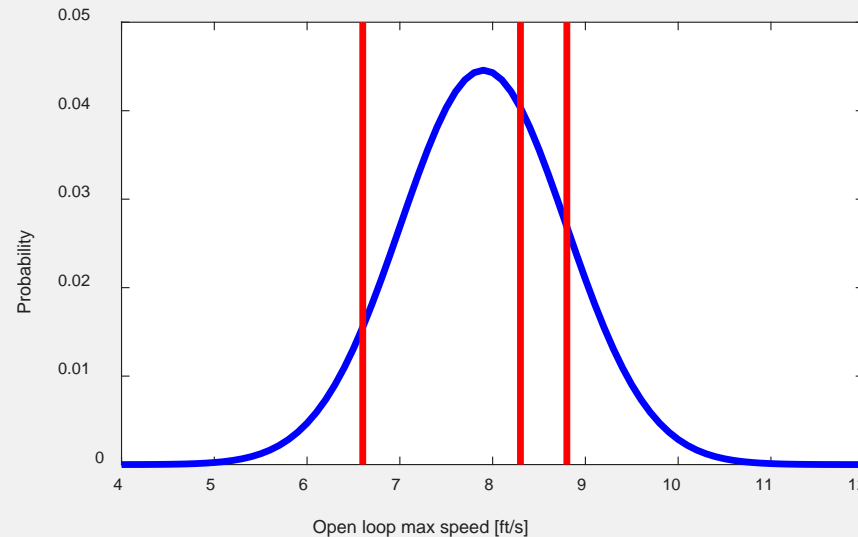
$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$



Probability Distributions

- Use Bayes theorem
- Start by assuming nothing
 - $P(s) = \text{uniform}$
 - $P(s | m) = P(m | s) * c_1/c_2$
 - Simplified: $P(s | m) = P(m | s)$
 - Example, what if the actual max speed is 11 ft/s?
 - $P(s = 11 | m = [6.6, 8.33, 8.8]) = P(m = [6.6, 8.33, 8.8] | s = 11)$
 - $P(m = 6.6 | s = 11) * P(m = 8.33 | s = 11) * P(m = 8.8 | s = 11)$

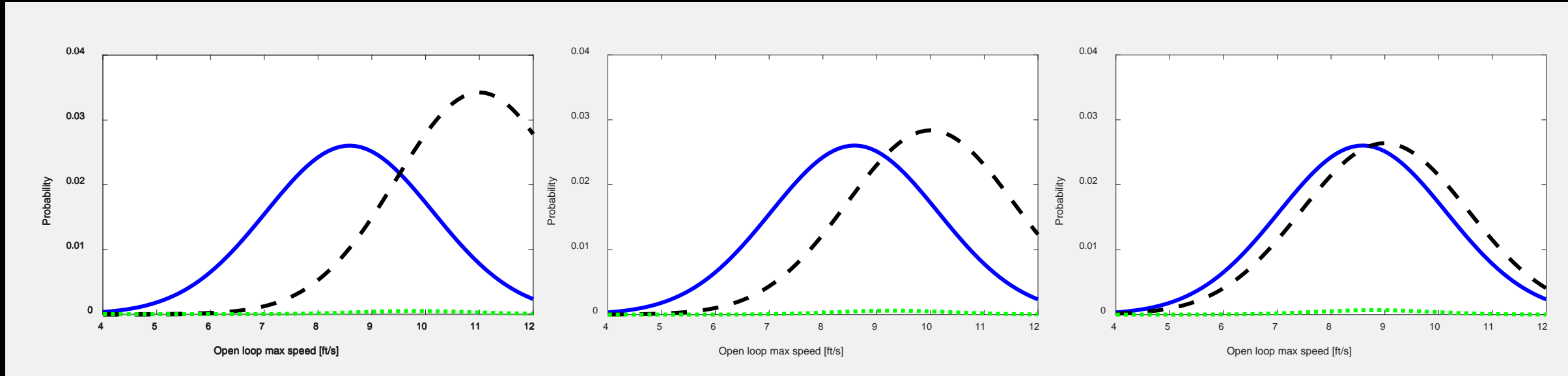
$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$



Probability Distributions

- Use Bayes theorem
 - Add a prior!
 - You know yesterday's speed, and you can kind of judge the current speed by eye
 - Prior: $7.91 \text{ ft/s} \pm 1.16 \text{ ft/s}$
 - $P(s = 11 \mid m = [6.6, 8.33, 8.8]) = P(m = [6.6, 8.33, 8.8] \mid s = 11) * P(s = 11)$
 $= P(m=6.6 \mid s=11) * P(s=11) * P(m=8.33 \mid s=11) * P(s=11) * P(m=8.8 \mid s=11) * P(s=11)$
- Repeat the process!

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$



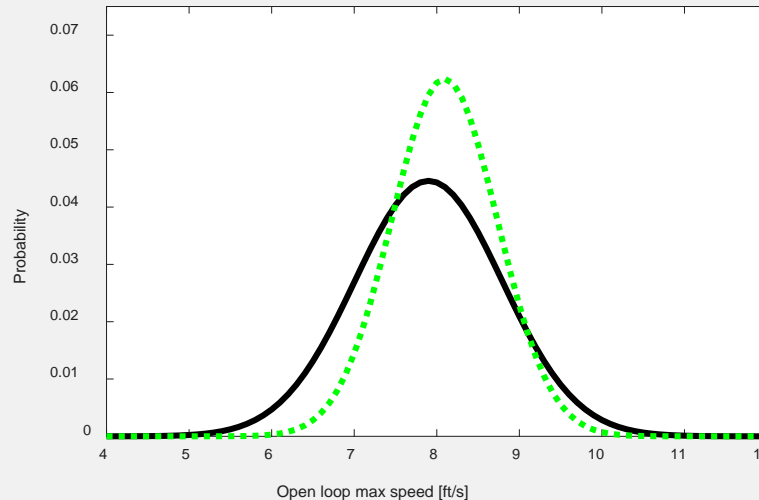
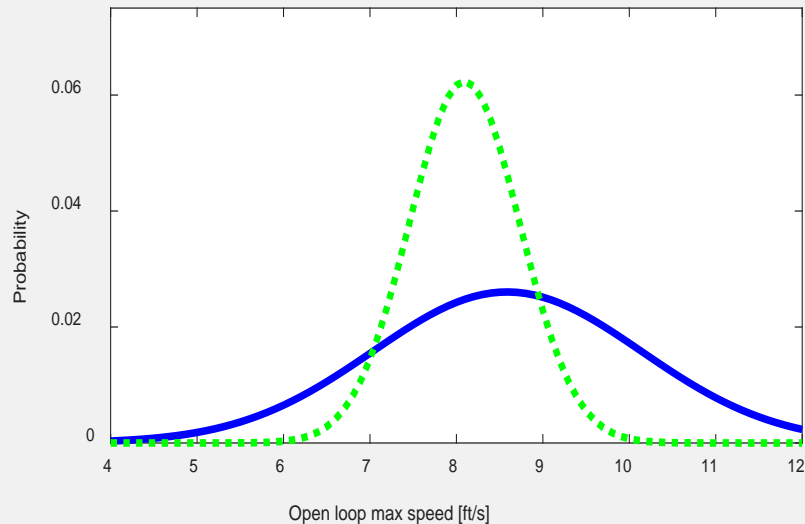
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Repeat the process!

Add everything up to get the posterior distribution

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$



Maximum A Posteriori
(MAP)

Probability Distributions

- Always believe the impossible, at least a little bit!
- Leave room for believing the unlikely. Leave a non-zero probability unless you are absolutely certain.
- “It ain’t what you don’t know that gets you into trouble. It’s what you know for sure that just ain’t so.” –Mark Twain
- “When you have excluded the impossible, whatever remains, however improbable, must be true.” Sherlock Holmes (Sir Arthur Conan Doyle)

Alice’s adventures in wonderland



References

- Probabilistic Robotics, book by *Dieter Fox, Sebastian Thrun, and Wolfram Burgard*
- How Bayes Theorem works (Youtube), by Brandon Rohrer