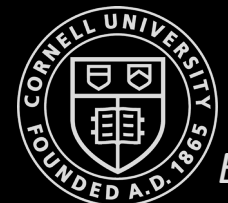


# Fast Robots

## Graph Search





## 1 Answer



Kirstin Petersen **STAFF**

3 hours ago



I'd be very sorry to see you drop the class after putting in all this effort to get so far!

In fact, our observations this weeks indicates that many students are still working on the PID lab, instead of the Kalman filter lab. Of course, you should do whatever you think is best for you, but first consider one of these options instead:

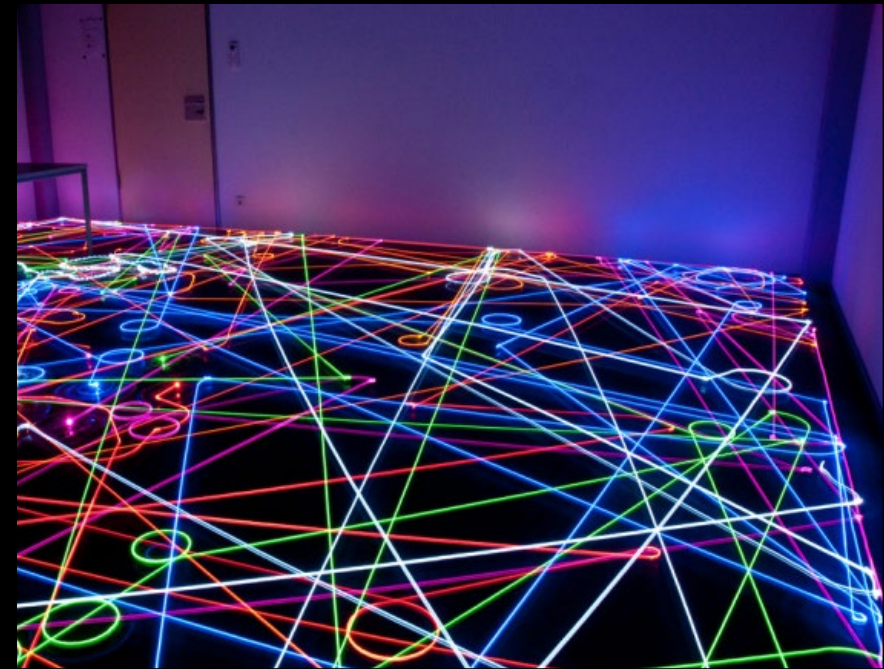
1. Skip the KF lab entirely. You loose 7.5 points, which is hardly the end of the world. You can still complete lab 8 either by 1) extrapolating from the TOF measurements you actually get, or 2) by trying to manually tune the distance at which you start your turn/do your flip.
2. Skip the KF lab for now, and use one of your 2 [unlimited time lab extensions](#) to do KF later in the semester, so that you still get back the 7.5 points. (Be sure to let us know if that is the case).
3. Do half the KF lab for half the points. This only involves a step response (to be done in the lab), and KF on the previous data set you got from your PID control.

With respect to the PID lab itself, I'm happy to give an extension on the write-up if you think that would help! We can also discuss switching to audit later if you want to give it a few more weeks of thought. I can always sign a petition to allow you to switch after the drop deadline.

Comment Edit Delete Endorse ...

# Algorithms and Search

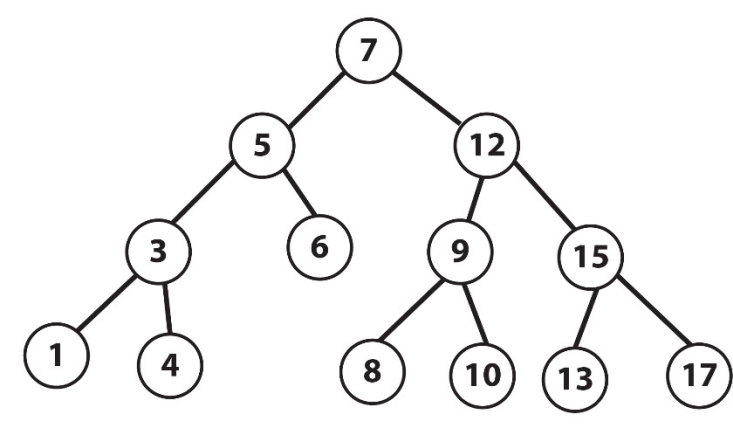
- **What is the simplest thing to do?**
  - Random or brute force search
- **Other methods?**
  - Uninformed search
    - Depth First Search (DFS)
    - Breadth First Search (BFS)
    - Dijkstra's Search
  - Informed Search
    - Greedy
    - A\*
    - (and many more)



# Comparing Search Algorithms

## Definitions

- Complete
  - A search algorithm is complete if, whenever at least one solution exists, the algorithm is guaranteed to find a solution within a finite amount of time
- Time complexity
  - The time complexity of a search algorithm is an expression for the worst-case amount of time it will take to run, expressed in terms of the maximum path length,  $m$ , and the maximum branching factor,  $b$
- Space complexity
  - The space complexity of a search algorithm is an expression for the worst-case amount of memory that the algorithm will use, expressed in terms of  $m$  and  $b$
- Optimality
  - A search is optimal if it is complete, and only returns cost-minimizing solutions

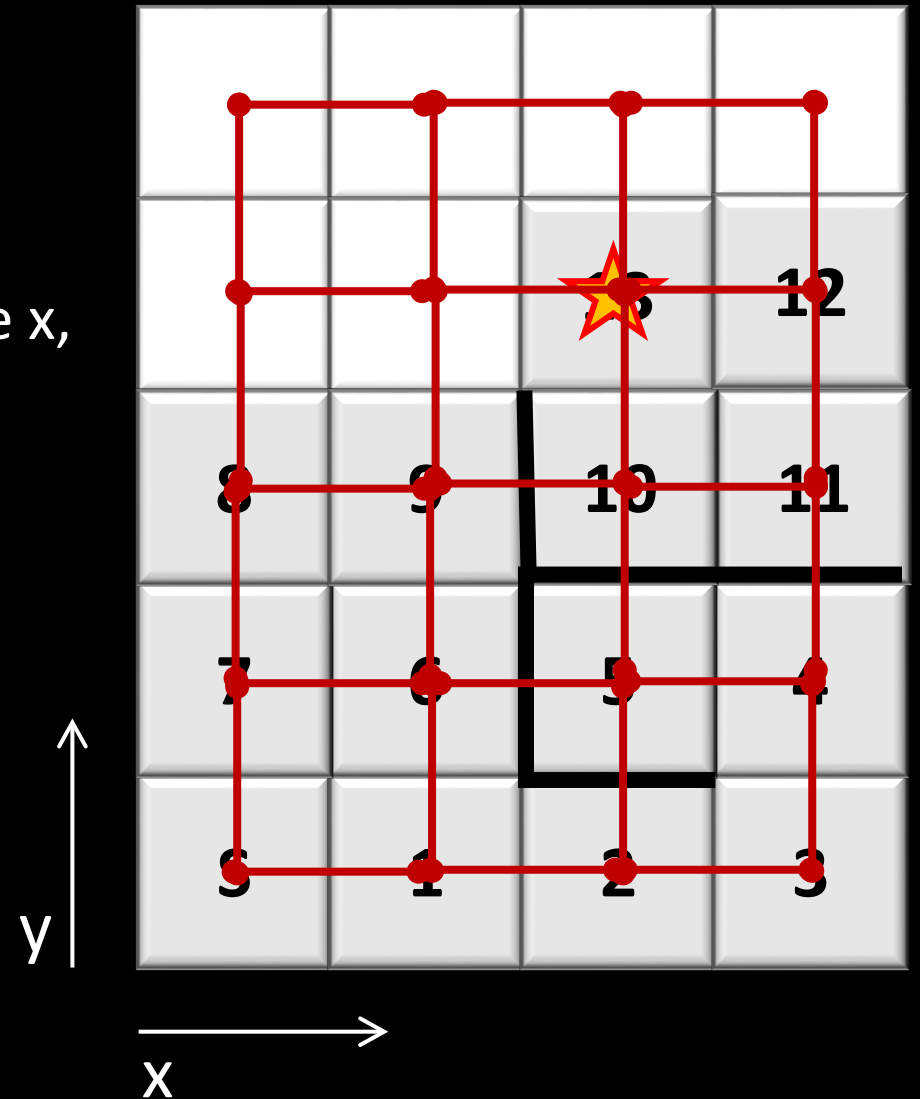


# Algorithms and Search

- **What is the simplest thing to do?**
  - Random or brute force search
- **How many grid traversals will brute force take?**
  - First establish a search order
  - Advance x first, then increment y and decrease x, etc.

Search order: N, E, S, W

Find a goal

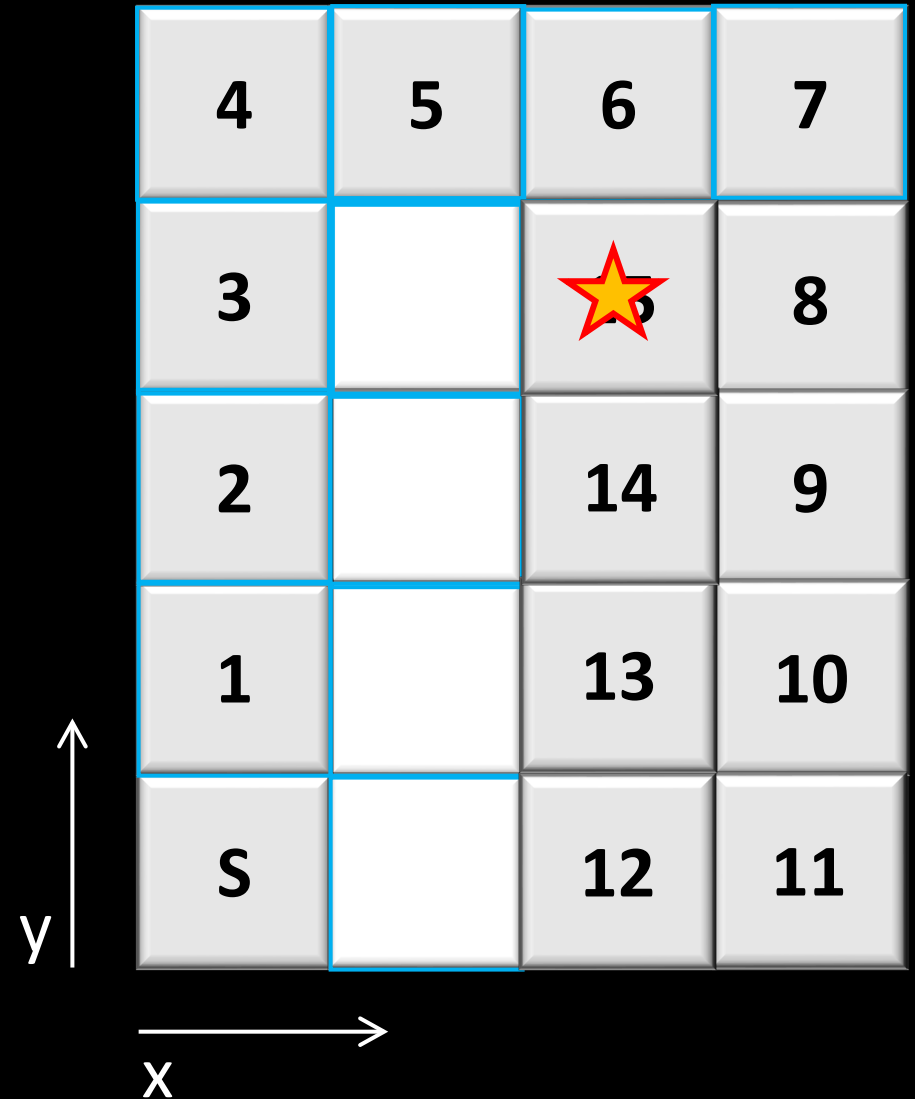


# Algorithms and Search

- **What is the simplest thing to do?**
  - Random or brute force search
- **Other methods?**
  - Depth First Search (DFS)

Search order: N, E, S, W

Find a goal

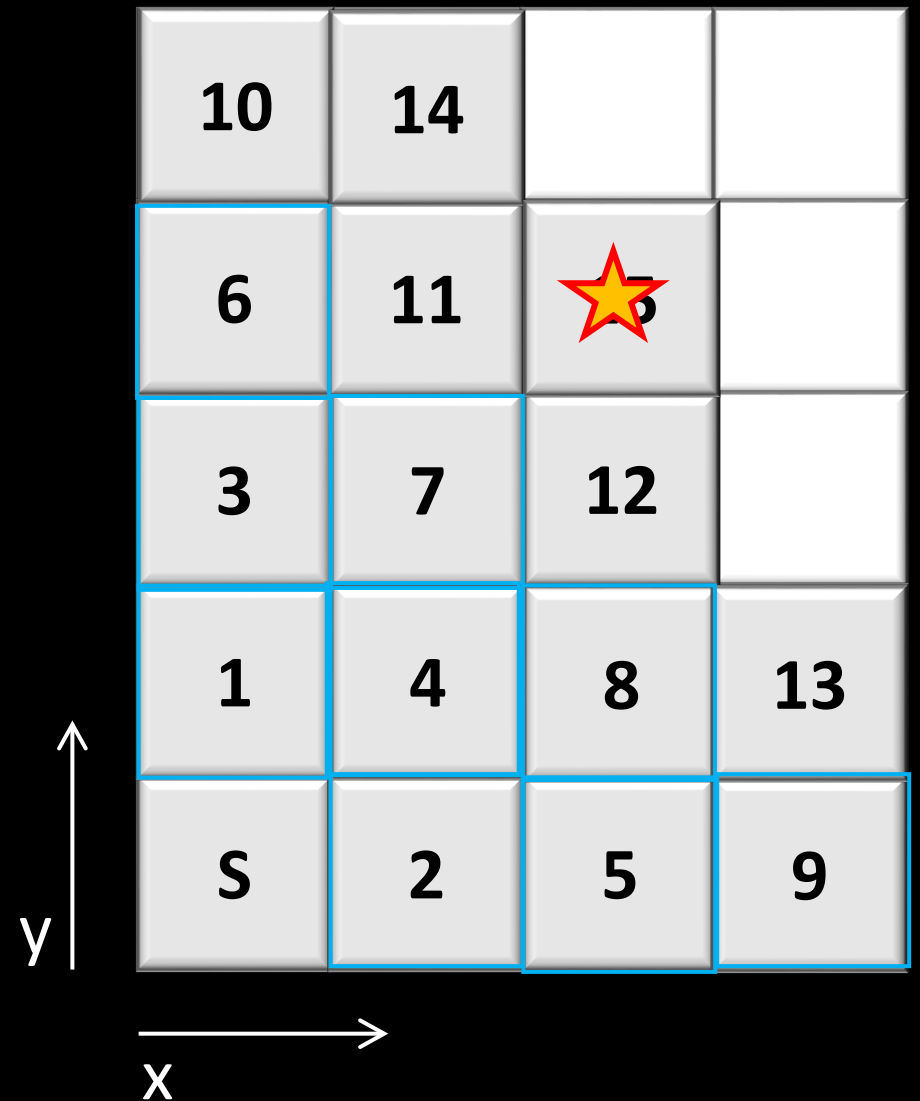


# Algorithms and Search

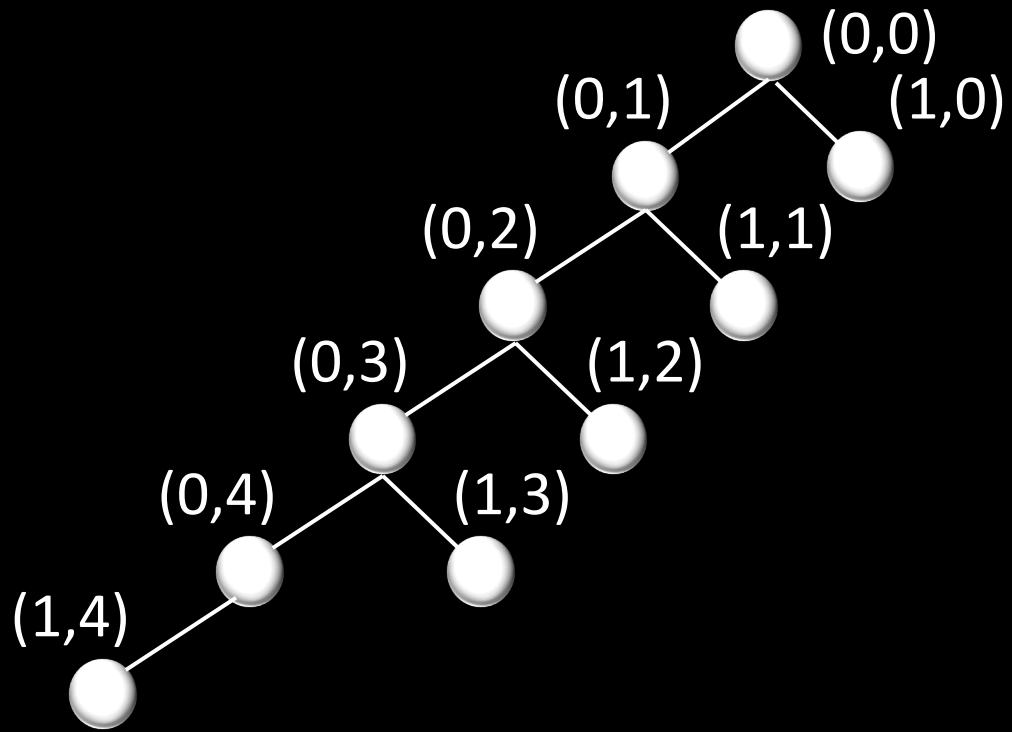
- **What is the simplest thing to do?**
  - Random or brute force search
- **Other methods?**
  - Depth First Search (DFS)
  - Breadth First Search (BFS)

Search order: N, E, S, W

Find a goal



# Depth First Search (DFS)



and so on...

Search order: N, E, S, W

Find a goal



y

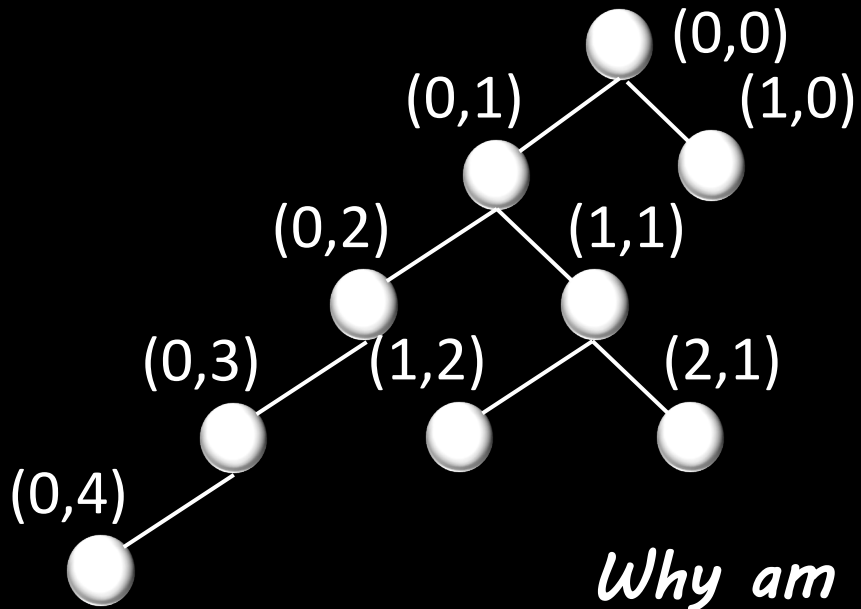
x



# Depth First Search (DFS)

Search order: N, E, S, W

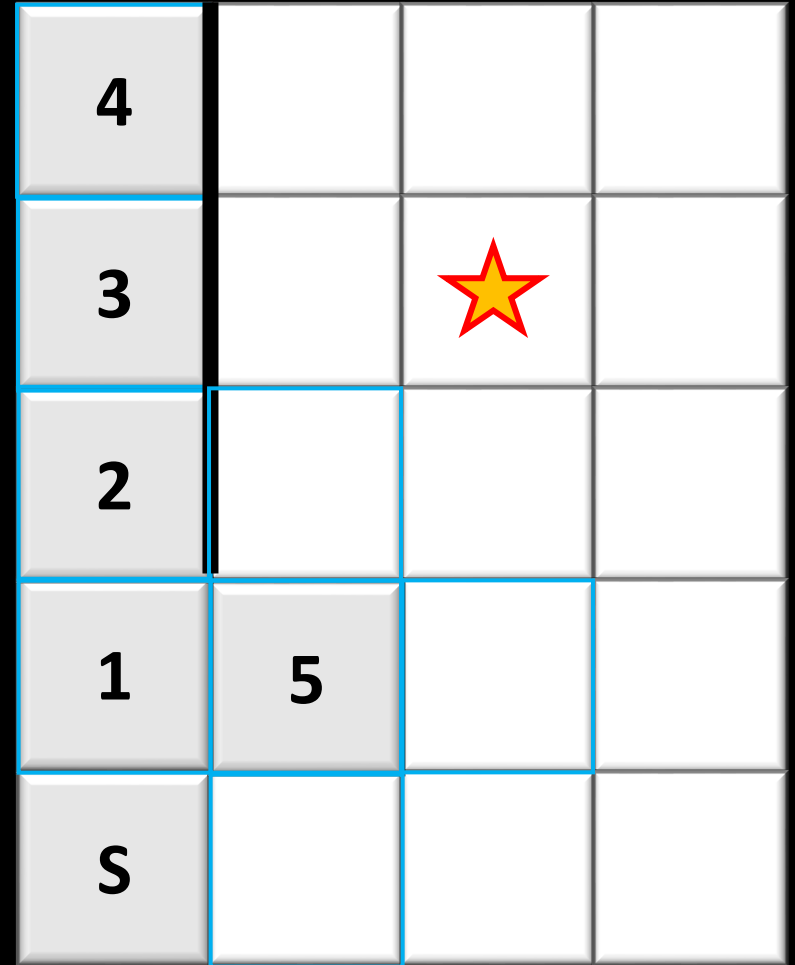
Find a goal



*Why am I not also adding (1,0)?*

- Keep track of what is already on the frontier

and so on...



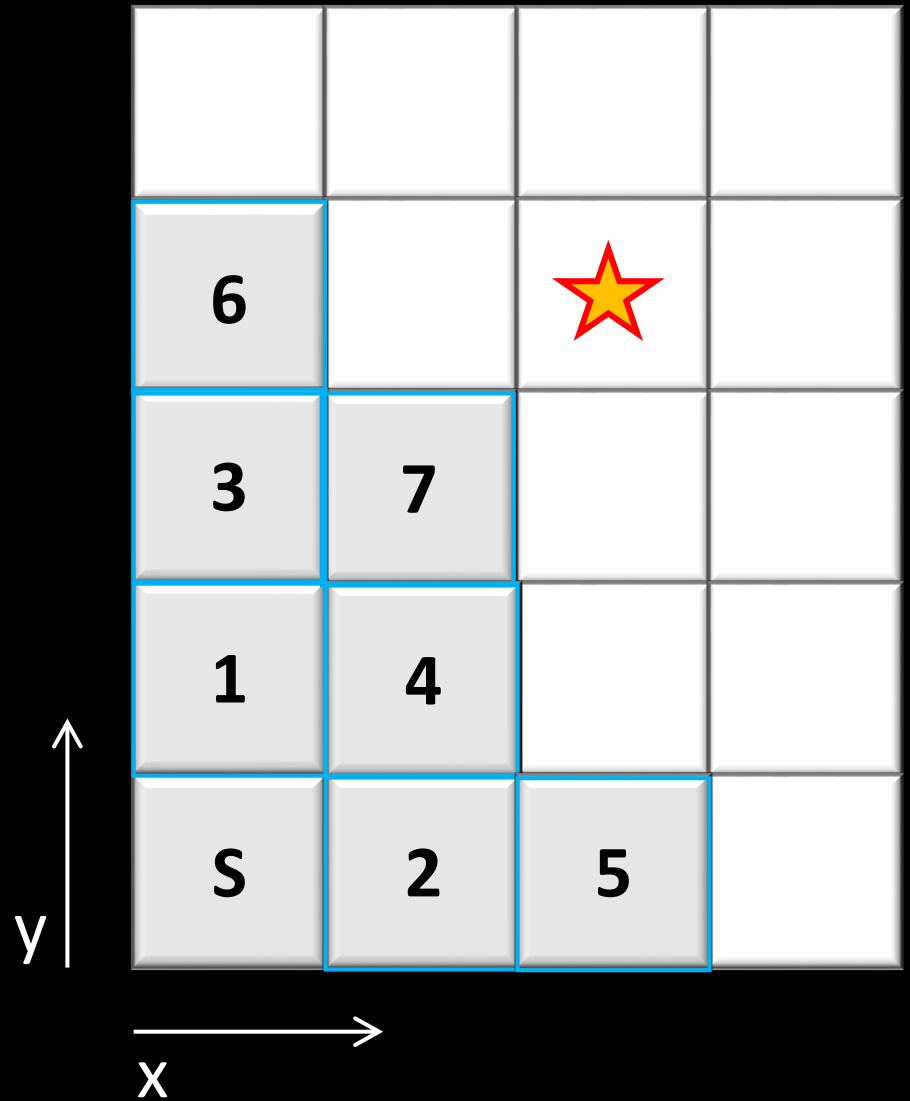
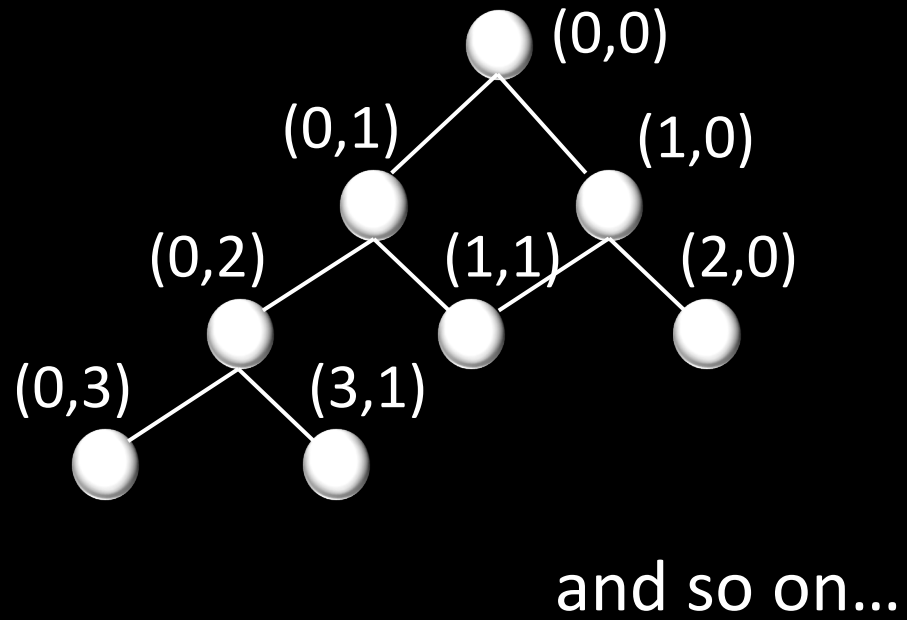
y ↑

x →

# Breadth First Search (BFS)

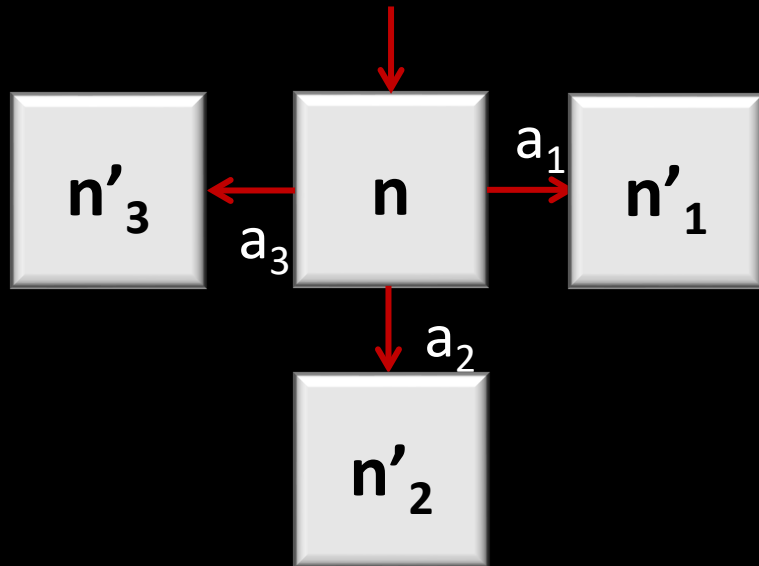
Search order: N, E, S, W

Find a goal

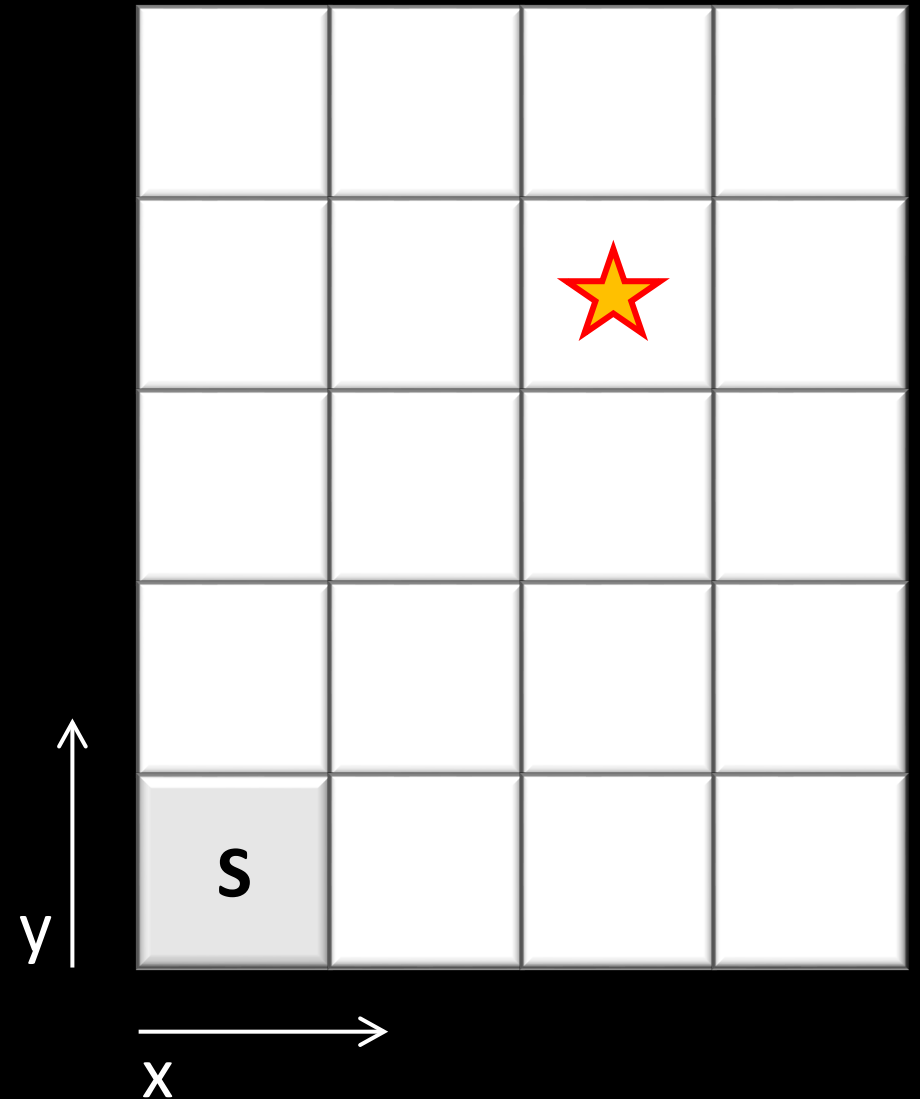


# Search Algorithms, General

- Common graph structure
  - For every node,  $n$
  - you have a set of actions,  $a$
  - that moves you to a new node,  $n'$



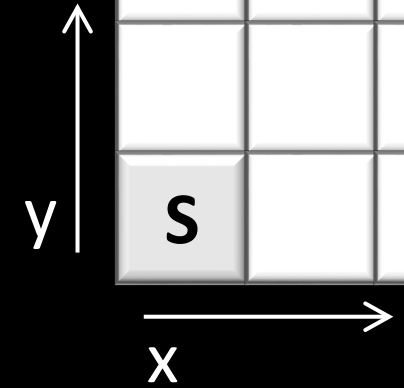
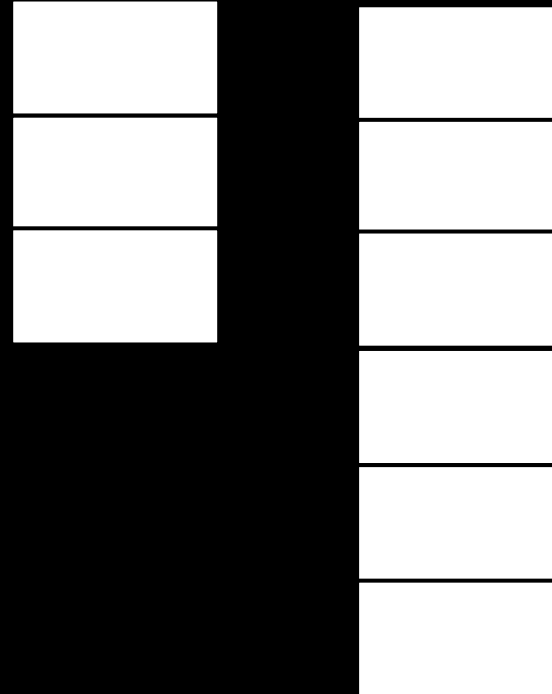
Find a goal



# Search Algorithms, General

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited



*How much space  
to allocate to  
visited?*

# Depth First Search (DFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited

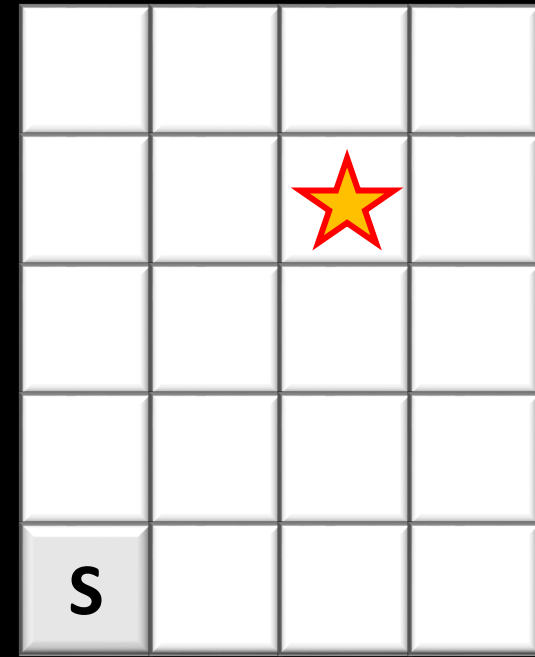
0,0

...

X\*Y

y ↑

→ x

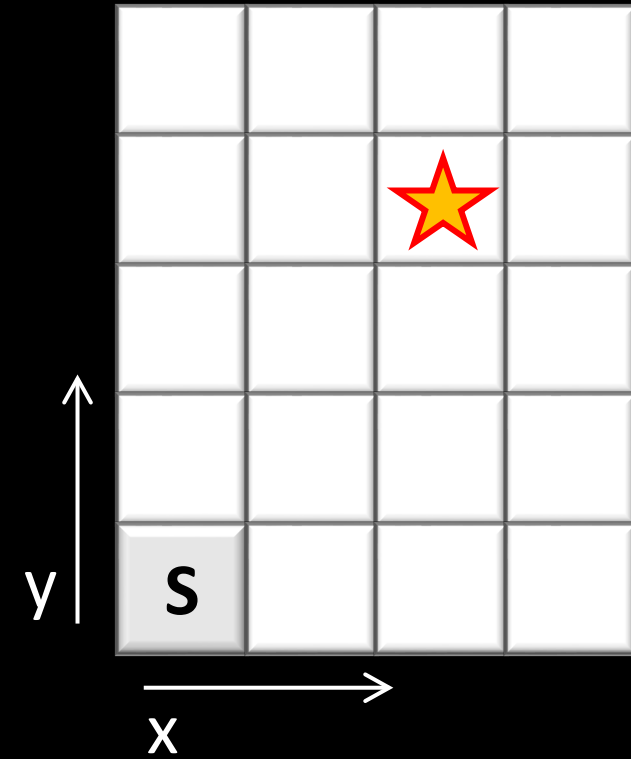


# Depth First Search (DFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited

0,1	0,0
1,0	
	...
	X*Y

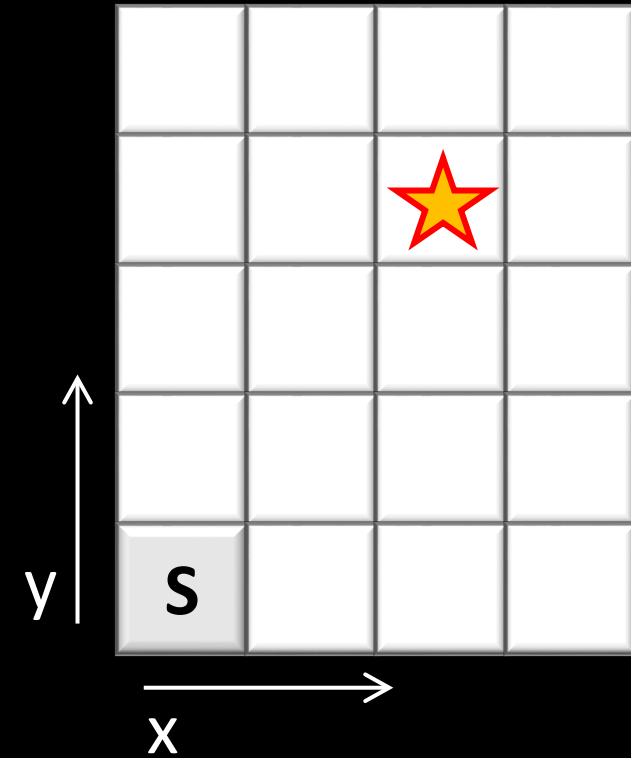


# Depth First Search (DFS)

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n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited

0,1	0,0
1,0	0,1
1,0	
	...
	X*Y

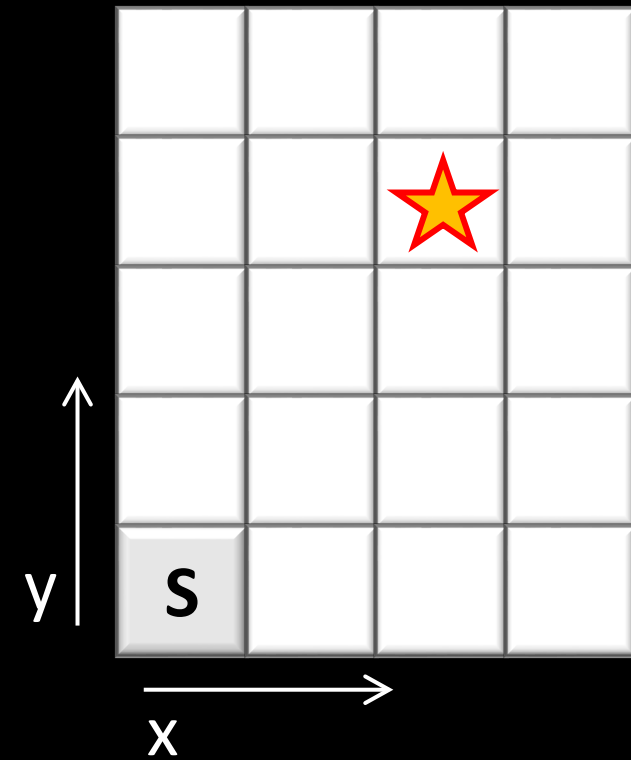


# Depth First Search (DFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited

0,2	0,0
1,1	0,1
1,0	0,2
	...
	X*Y



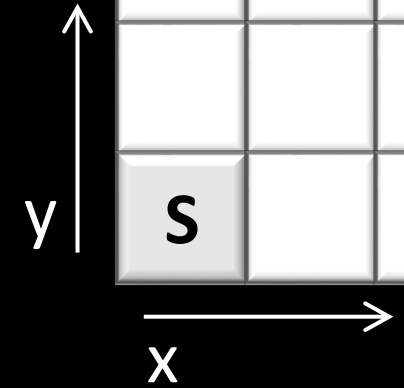


# Depth First Search (DFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier      visited

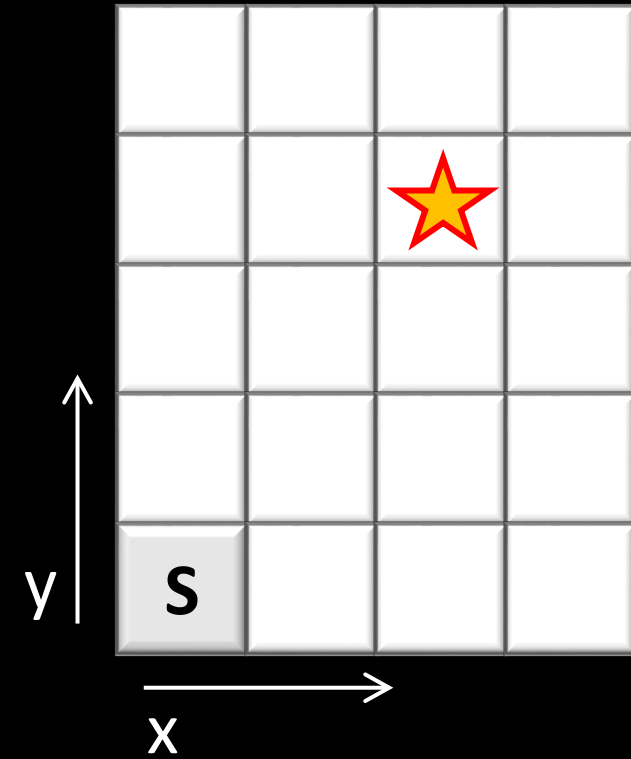
0,3	0,0
1,2	0,1
1,0	0,2
1,0	0,3



# Depth First Search (DFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  append n to visited
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

frontier	visited
0,4	0,0
1,3	0,1
1,1	0,2
1,0	0,3
1,0	
etc...	

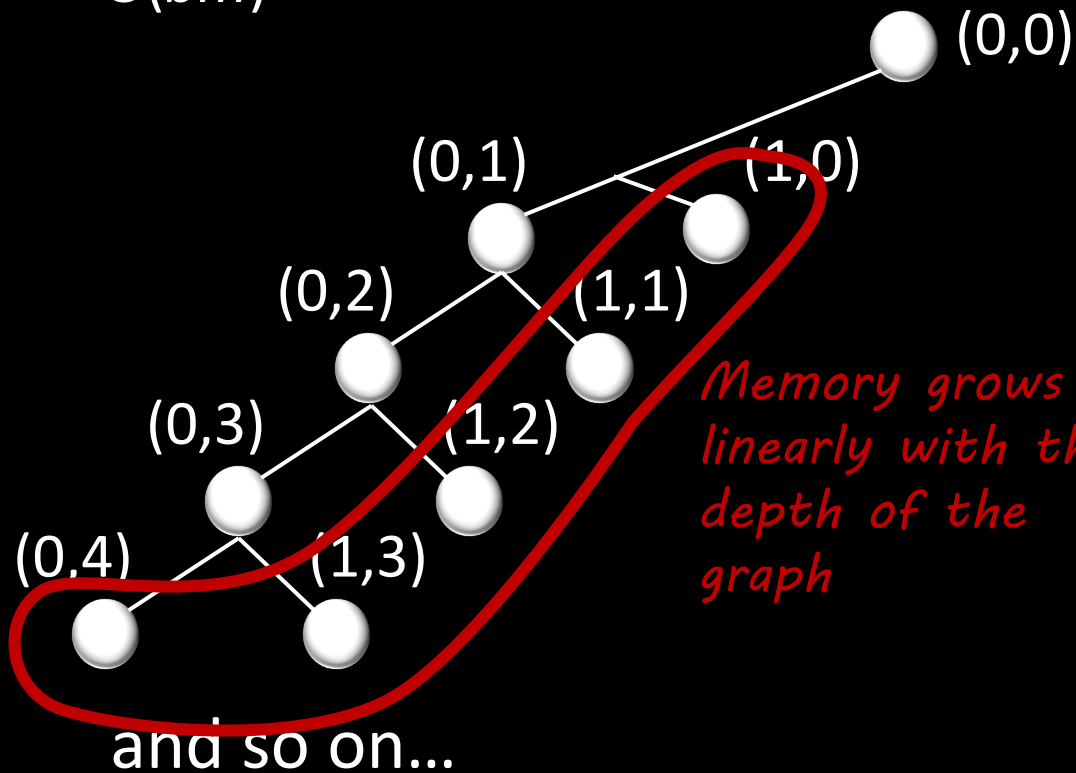


**Frontier Buffer?**

- *Last-In First-Out (LIFO) Buffer*

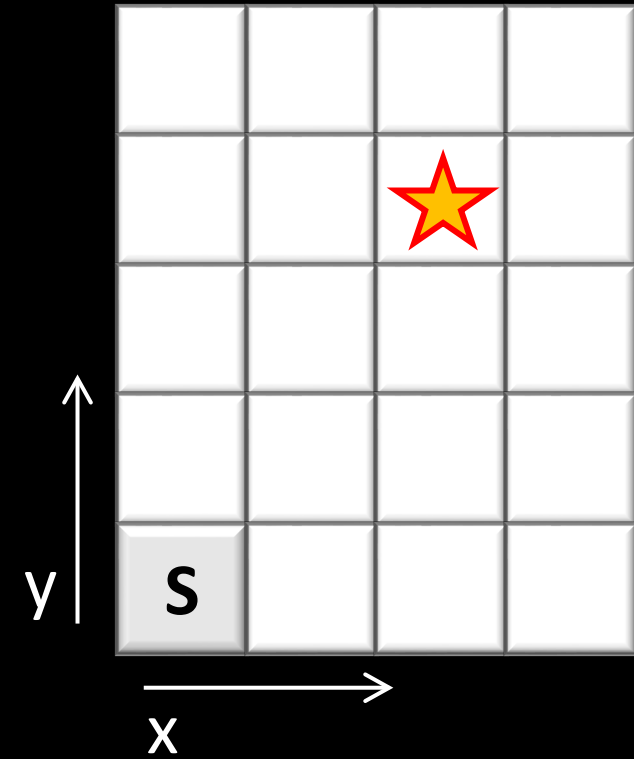
# Depth First Search (DFS)

- Is it complete?
  - Yes, but only on finite graphs
- What is the time complexity?
  - $O(b^m)$
- What is the space complexity?
  - $O(bm)$



*Memory grows linearly with the depth of the graph*

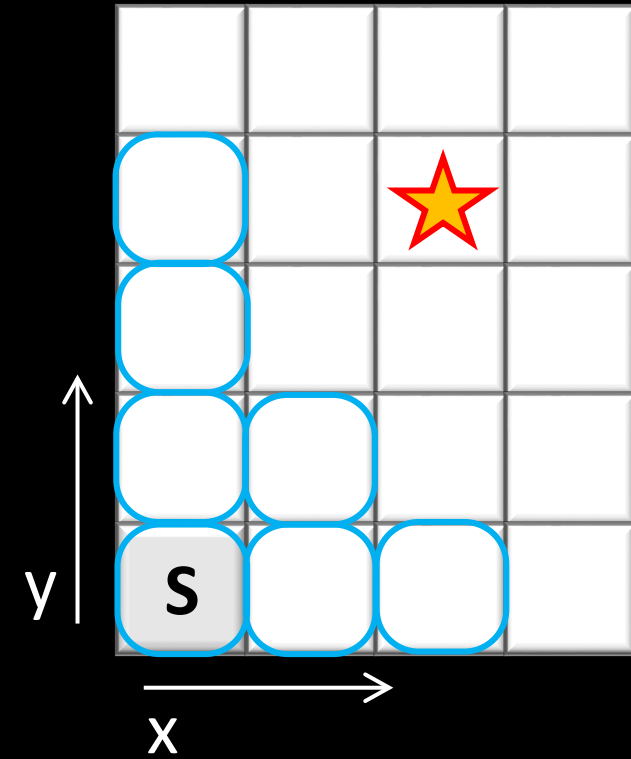
frontier	visited
0,4	0,0
1,3	0,1
1,2	0,2
1,1	0,3
1,0	...
	X*Y



# Breadth First Search (BFS)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  if n is goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      append n' to frontier
```

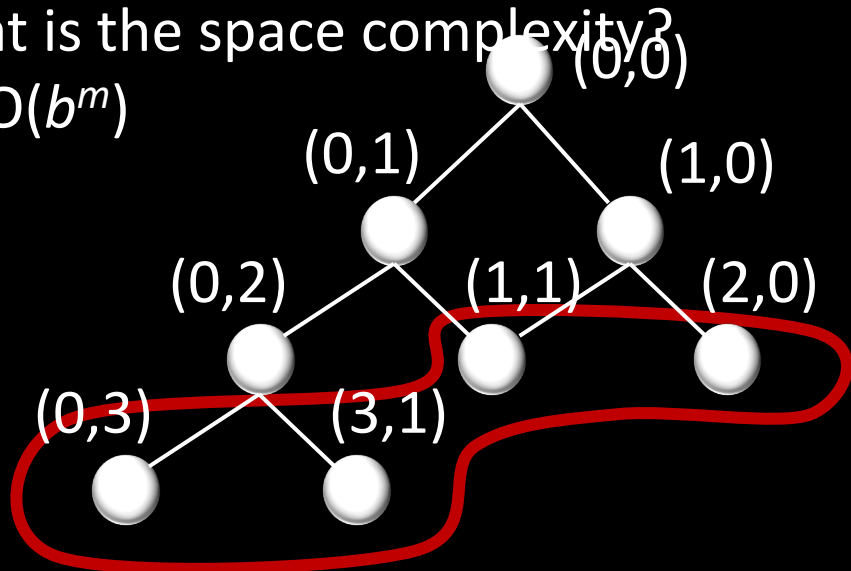
frontier	visited
0,0	0,0
0,1	0,1
1,0	1,0
0,2	0,2
1,1	
2,0	...
0,3	



*Type of Buffer?*  
*First-In First-Out (FIFO) Buffer*

# Breadth First Search (BFS)

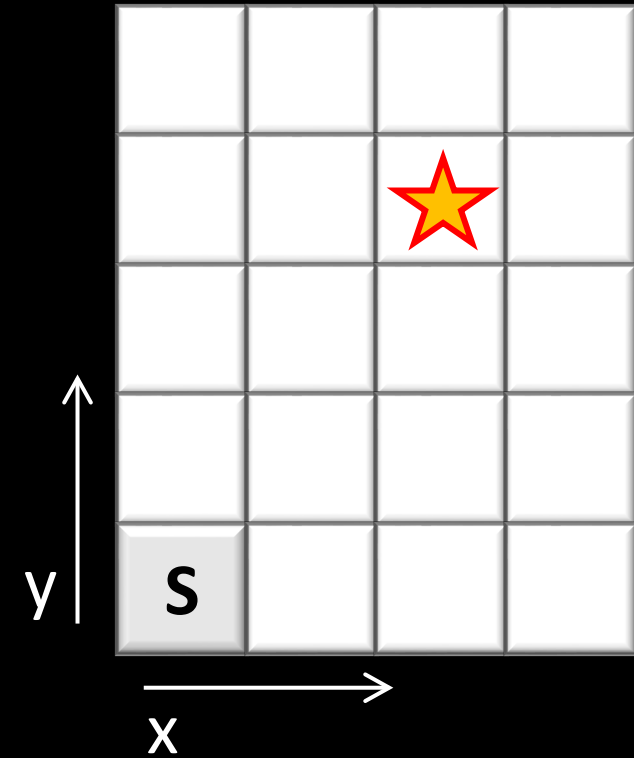
- Is it complete?
  - Yes, as long as  $b$  is finite
- Is it optimal?
  - Yes
- What is the time complexity?
  - $O(b^m)$
- What is the space complexity?
  - $O(b^m)$



*Memory grows exponentially with the depth of the graph*

frontier      visited

	0,0
	0,1
	1,0
	0,2
1,1	
2,0	
0,3	
	...

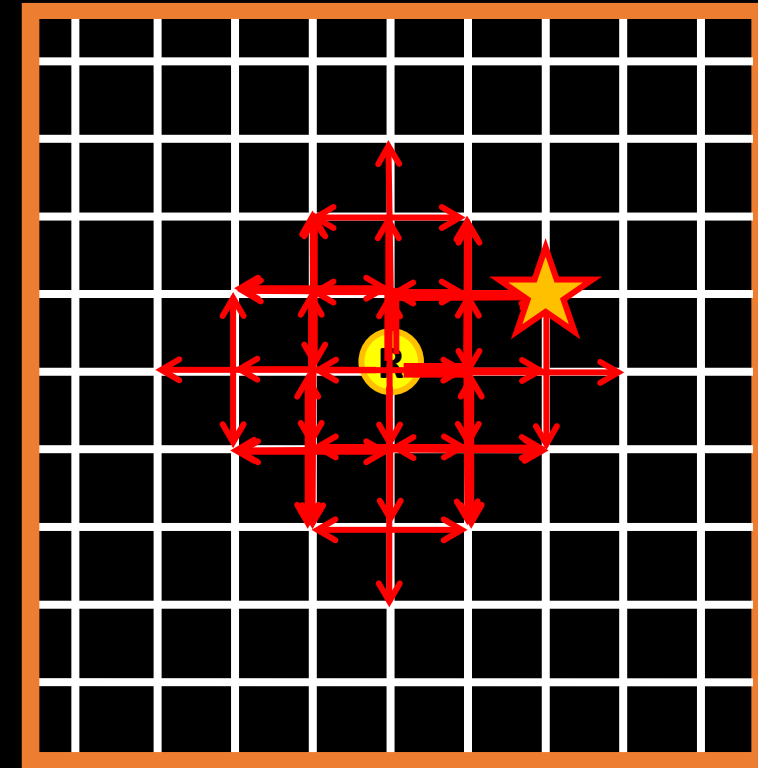
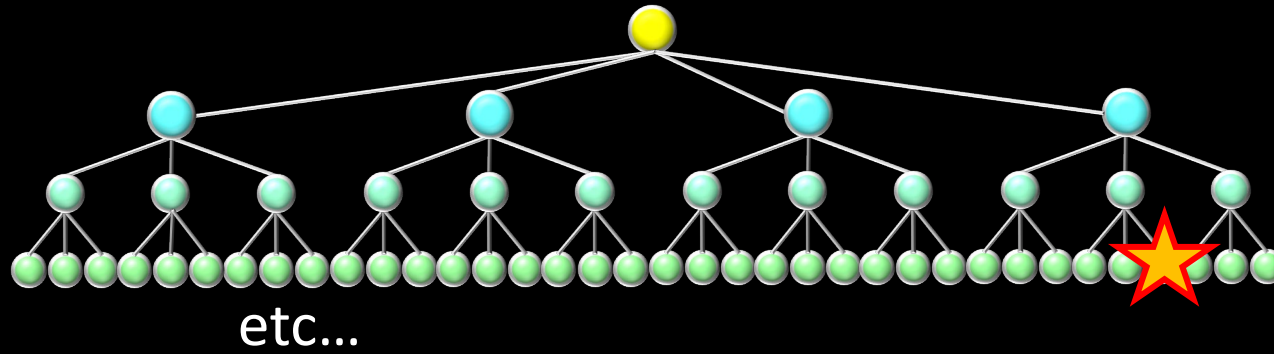


*Type of Buffer?  
First-In First-Out (FIFO) Buffer*

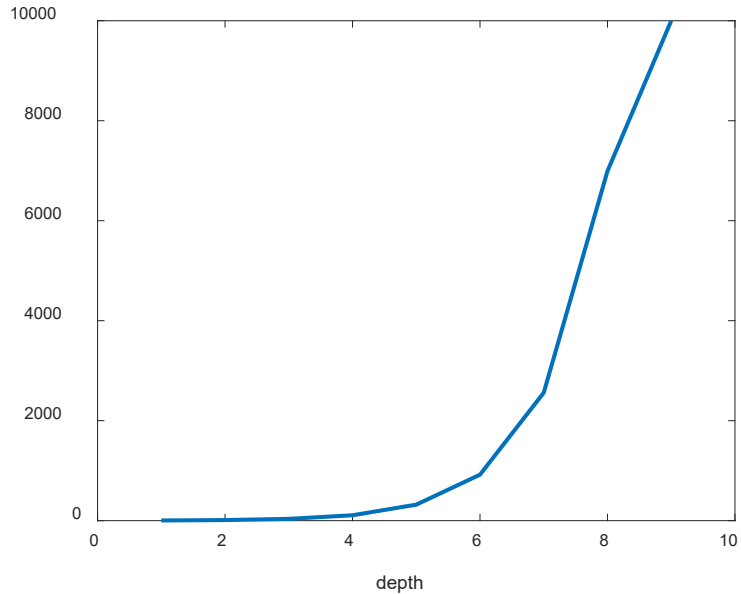
# BFS: Memory and Computation

Frontier size:

- 4
- 12
- 36



BFS: Growth of frontier

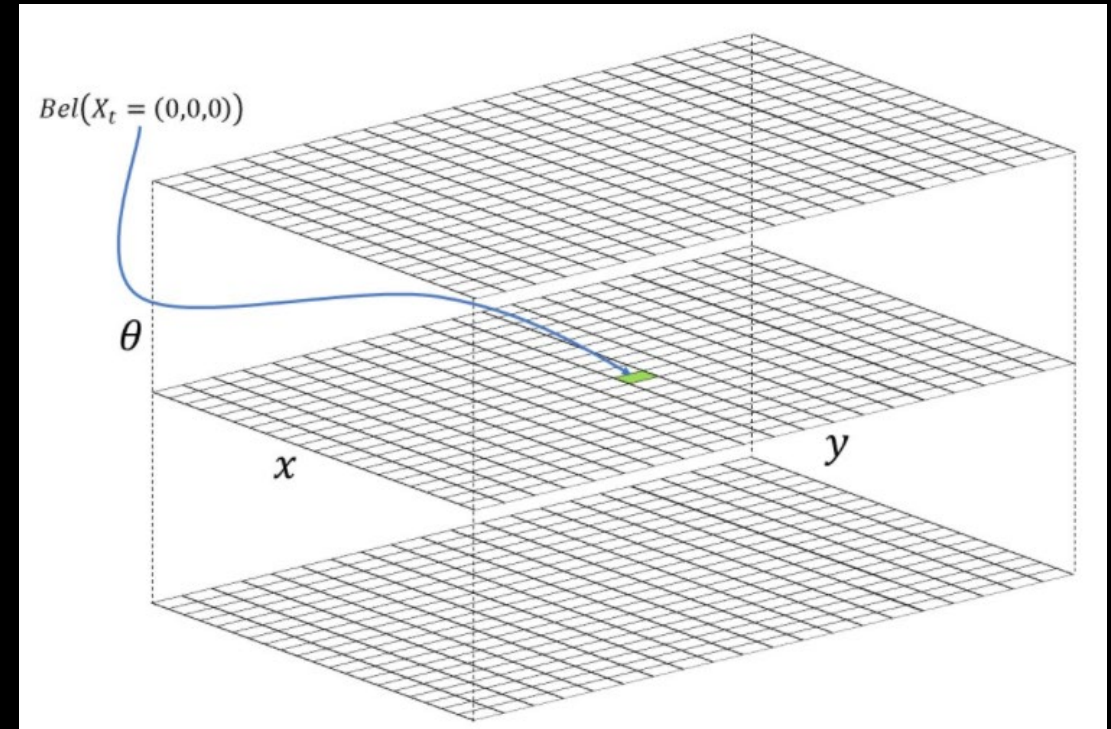


# Uninformed Search Algorithms, General

- When is DFS appropriate?
  - If the memory is restricted
  - If solutions tend to occur at the same depth in the tree
- When is DFS inappropriate?
  - If some paths have infinite length / if the graph contains cycles
  - If some solutions are very deep, while others are very shallow
- When is BFS appropriate?
  - If you need to find the shortest path
  - If memory is not a problem
  - If some solutions are shallow
  - If there might be infinite paths
- When is BFS inappropriate?
  - If memory is limited / if the branching factor is very large
  - If solutions tend to be located deep in the tree

# ECE4960 Robot

- Is BFS / DFS possible for your task on the Artemis?
  - What is the maximum branching factor?
    - $b = 4$
  - What is the longest path?
    - $m \sim 20 \times 20 = 400$
  - Depth First Search
    - Frontier:  $O(bm) = 1,600$  nodes
    - Float  $\rightarrow$  6.4kB
    - Artemis memory?
      - 1MB flash and 384k RAM
  - Breadth First Search
    - Frontier:  $O(b^m) = 4^{20 \times 20} = 6.7e240$  nodes

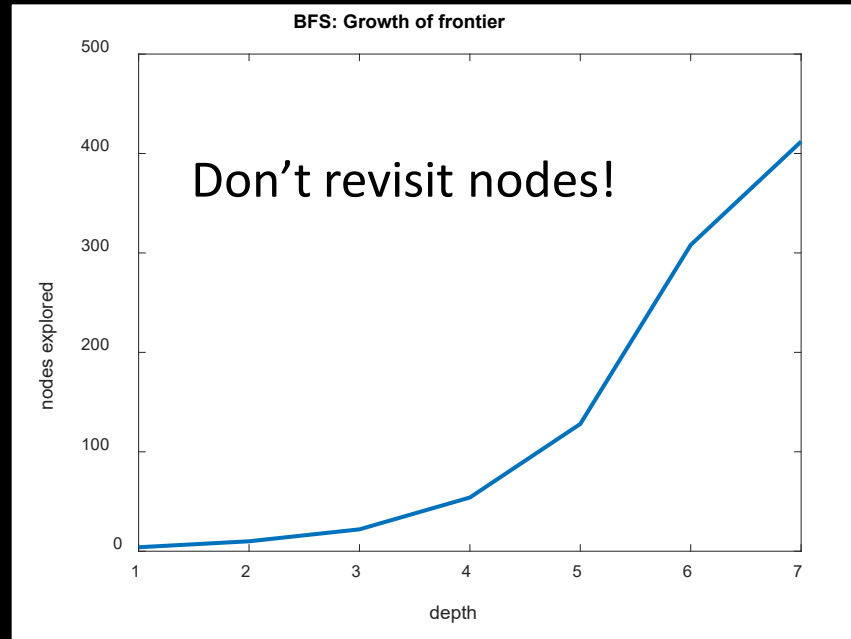
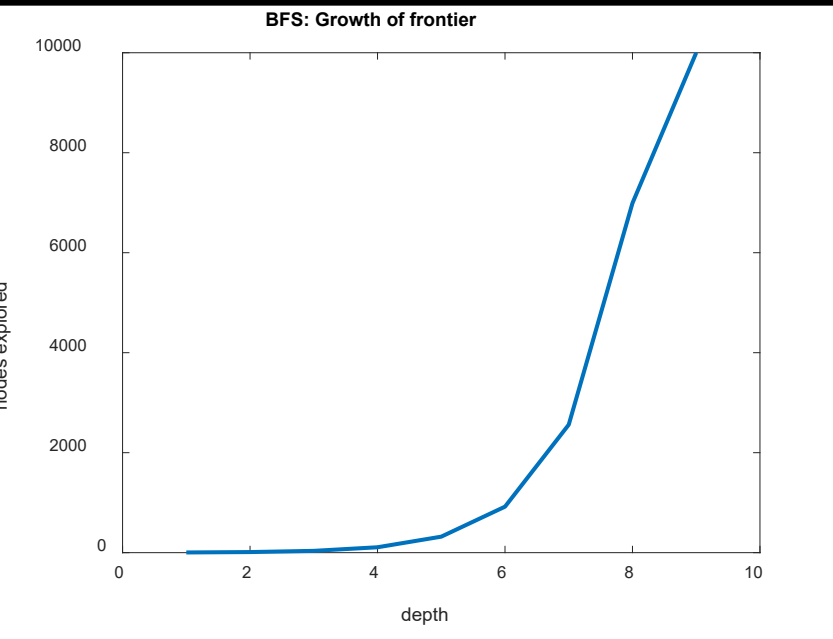
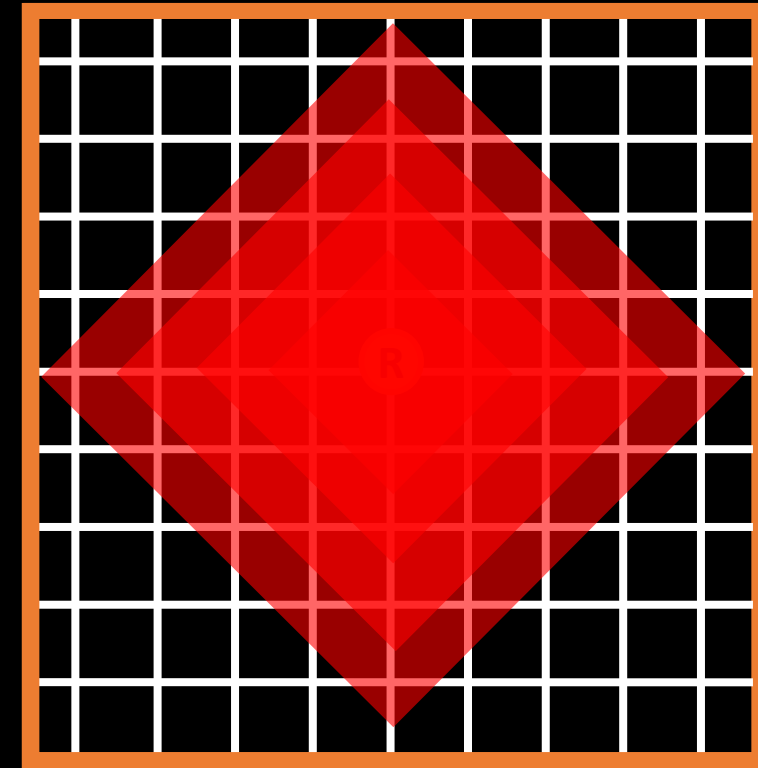
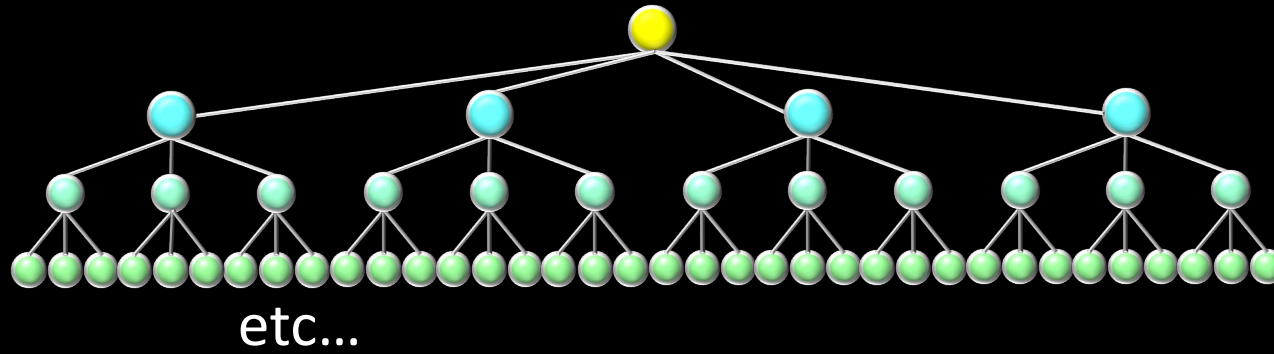




# BFS: Memory and Computation

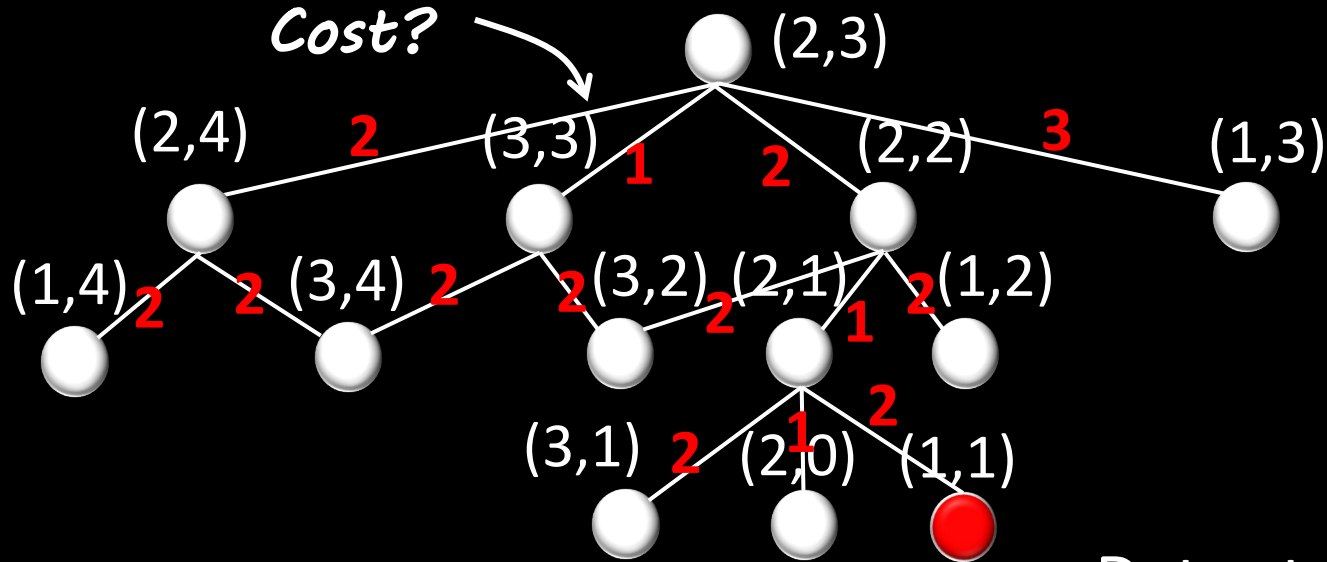
Frontier size:

- 4
- 12
- 36



# Lowest-Cost First Search

- Consider parent cost!



*What node to expand next?*

Data structure

- n.state
- n.parent
- n.cost
- n.action

*What cost heuristic could we add?*

- Go straight, cost 1
- Turn one quadrant, cost 1

	(1,4)	(2,4)	(3,4)
	(1,3)	<del>R</del> →	(3,3)
	(1,2)	(2,2)	(3,2)
	G ←	(2,1)	(3,1)
		(2,0)	

# Lowest-Cost First Search

- Consider parent cost!

```
n = state(init)
frontier.append(n)
while(frontier not empty)
    n = pull state from frontier
    visited.append(n)
    if n = goal, return solution
    for all actions in n
        n' = a(n)
        if n' not visited
            priority = heuristic(goal,n')
            frontier.append(priority)
```

*What cost heuristic could we add?*

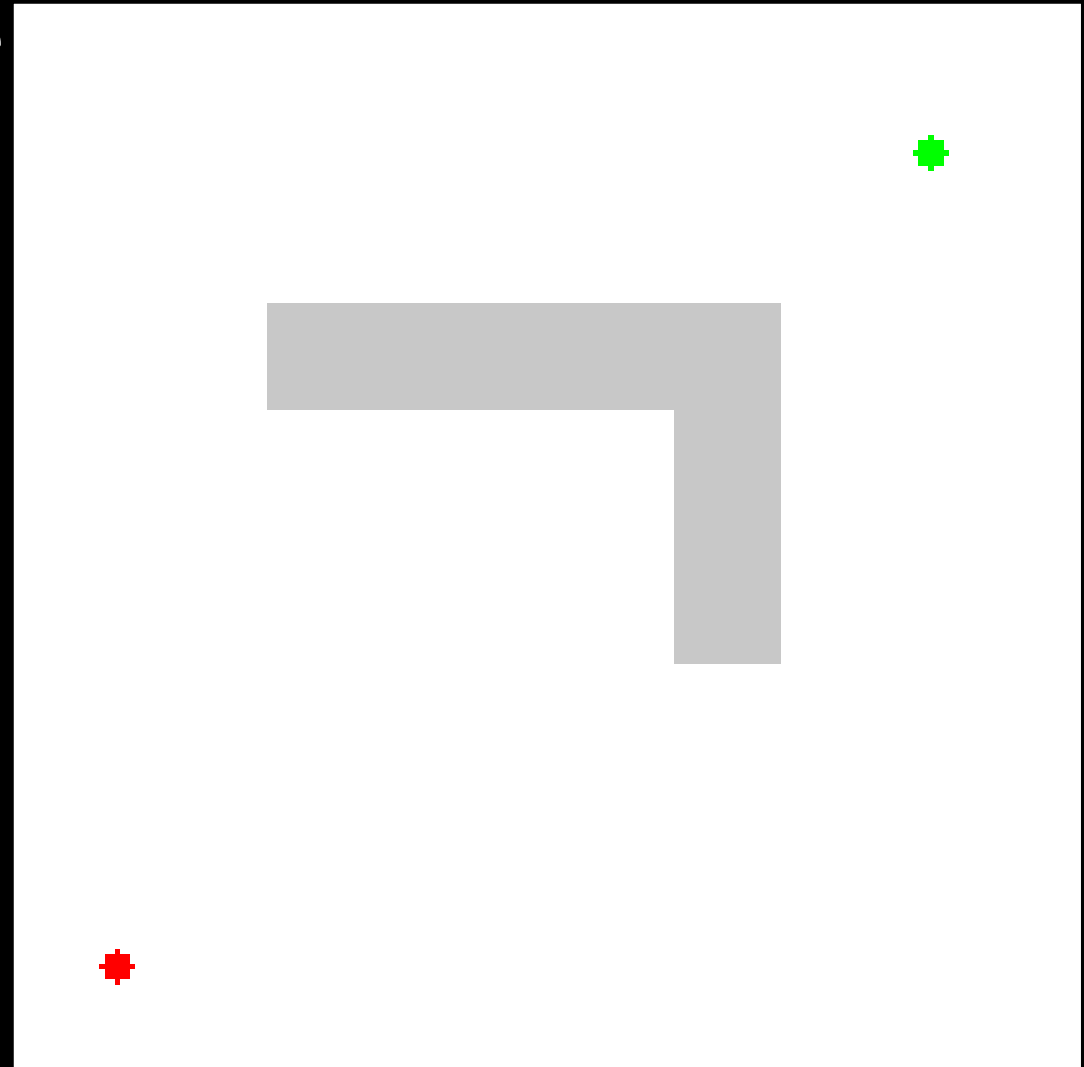
- Go straight, cost 1
- Turn one quadrant, cost 1

	(1,4)	(2,4)	(3,4)
	(1,3)	<del>R</del> →	(3,3)
	(1,2)	(2,2)	(3,2)
	G ←	(2,1)	(3,1)
		(2,0)	

# Lowest-Cost First Search

- Is it *complete*?
  - Yes, as long as path costs are positive
- What is the time complexity?
  - $O(b^m)$
- What is the space complexity?
  - $O(b^m)$

(Wikipedia)

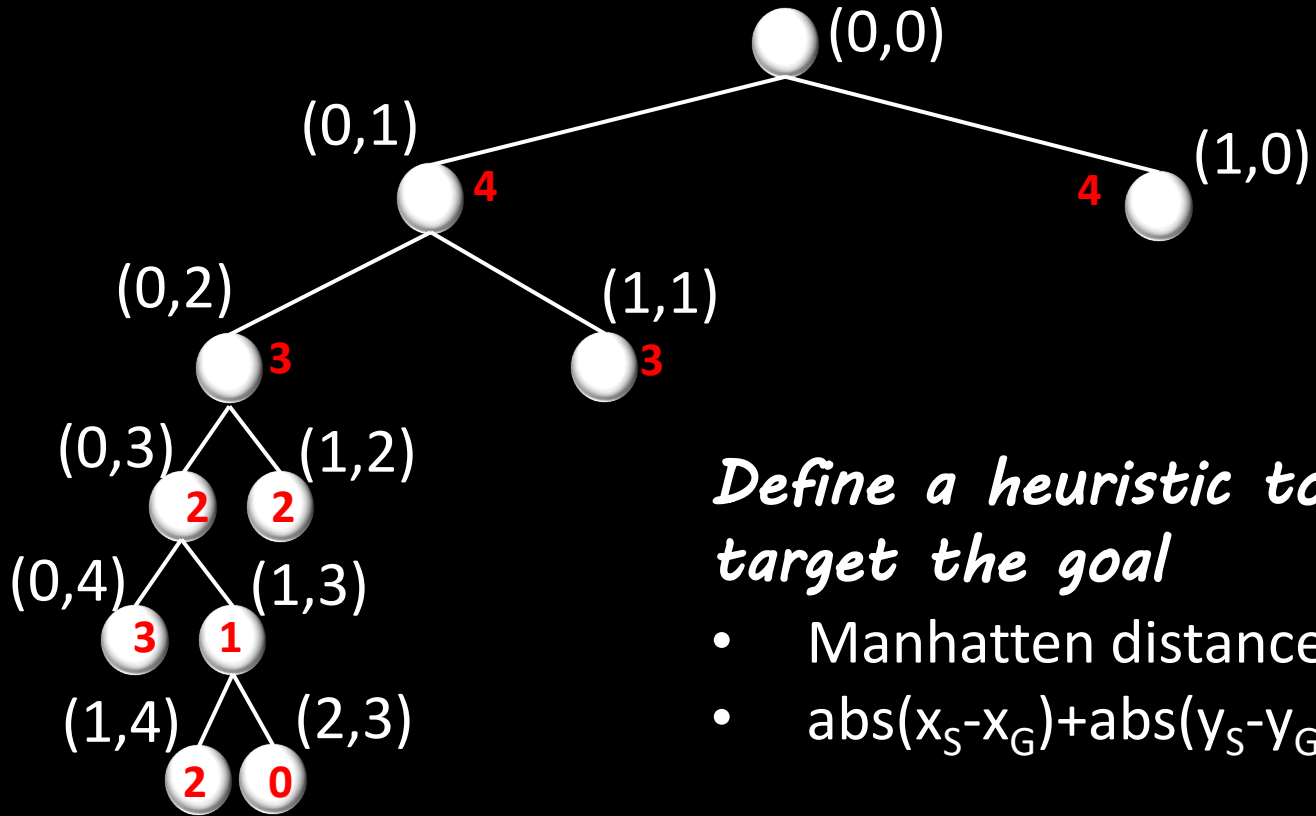


# Could we be smarter?

- *Sure, you know the graph and you know the goal is!*
- ...Informed search
  - Consider parent cost, and..
  - ..estimate the shortest path to the “goal”
- Assign a value to the frontier
  - Pick frontier closest to the goal (minimize distance)

# Informed Search

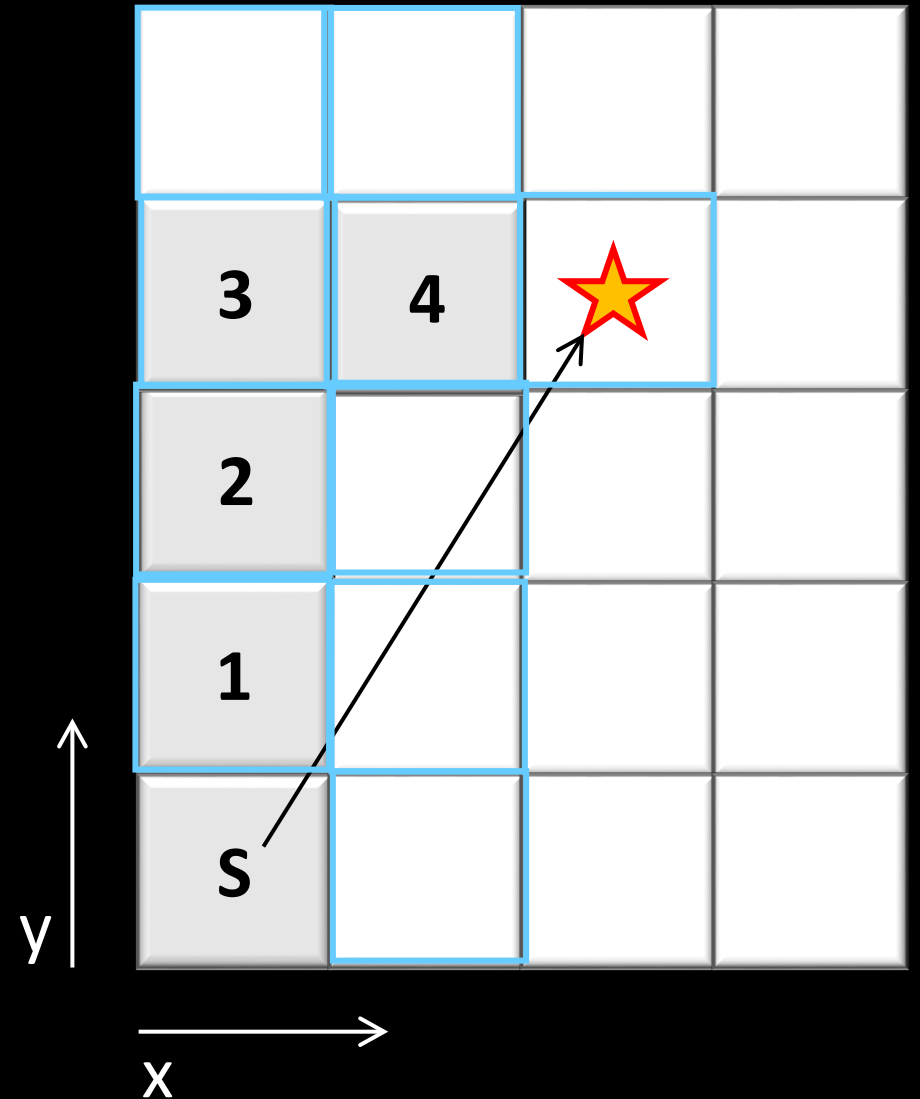
- Greedy Search



*Define a heuristic to target the goal*

- Manhattan distance
- $\text{abs}(x_S - x_G) + \text{abs}(y_S - y_G)$

Search order: N, E, S, W

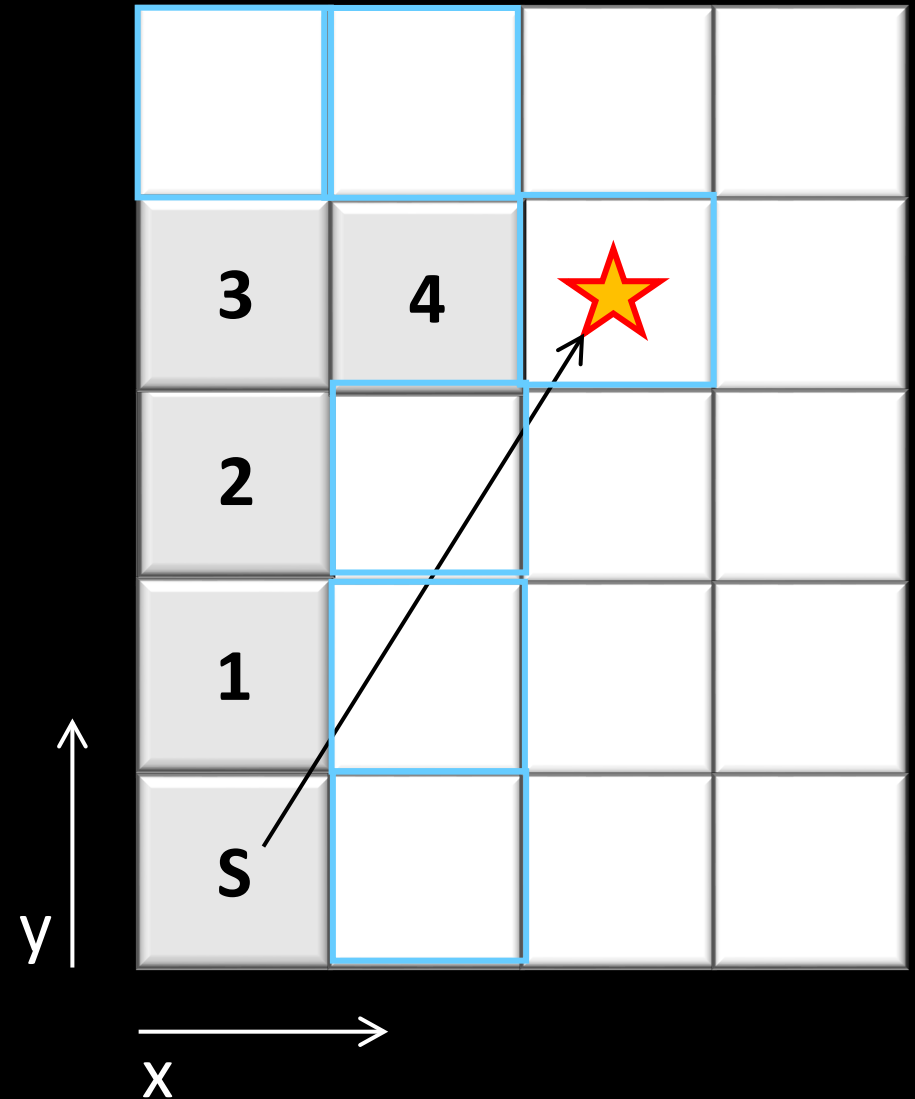


# Informed Search

Search order: N, E, S, W

- Greedy Search

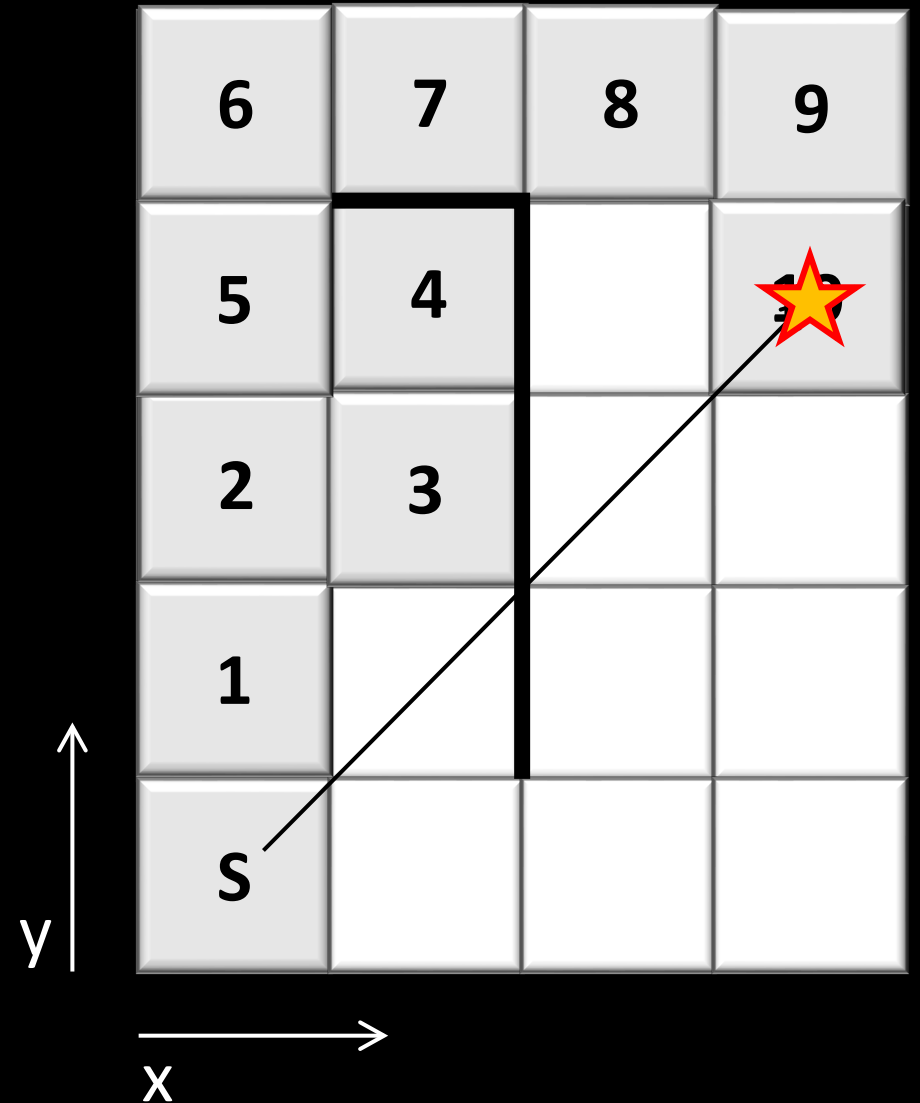
```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  visited.append(n)
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if n' not visited
      priority = heuristic(goal,n')
      frontier.append(priority)
```



# Informed Search

- Greedy Search
  - Complete?
    - No
  - Time complexity?
    - $O(b^m)$
  - Space complexity?
    - $O(b^m)$
  - Optimal?
    - no...

Search order: N, E, S, W





# Search Algorithms, General

- Breadth First Search
  - *Complete* and optimal
  - ...but searches *everything*
- Lowest-Cost First Algorithm *Considers parent cost*
  - *Complete and optimal*
  - ...but it wastes time exploring in directions that aren't promising
- Greedy Search *Considers goal*
  - *Complete (in most cases)*
  - ...only explores promising directions

*Can we do better?*

*A\**

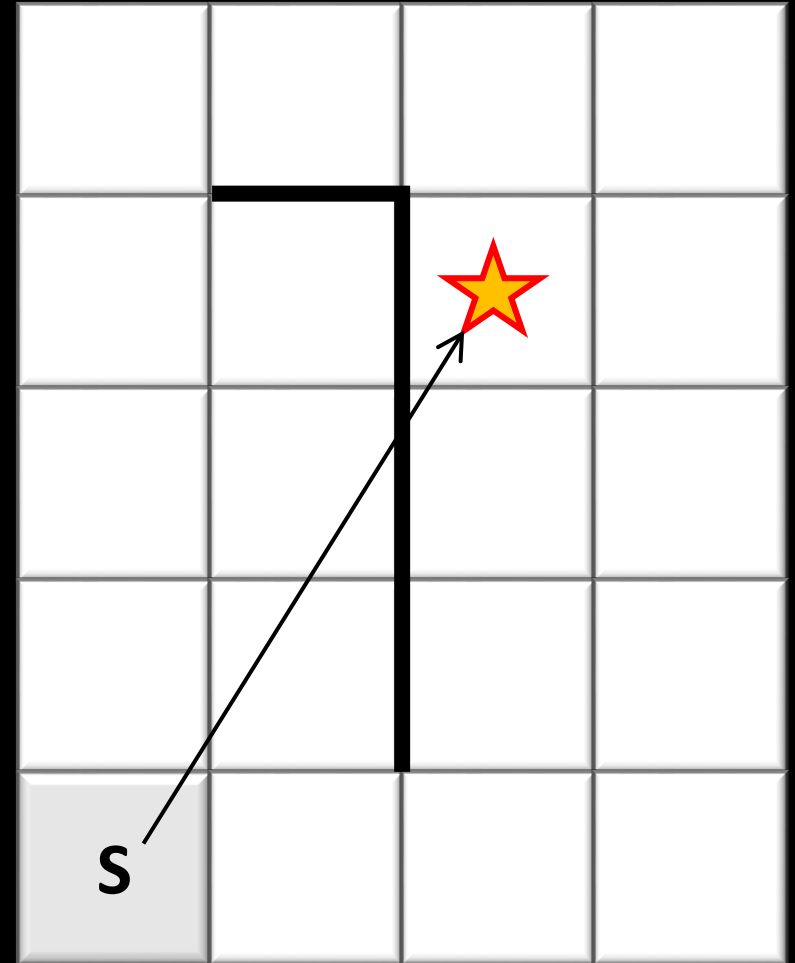
# Informed Search

- A\* (“A-star”)

```
n = state(init)
frontier.append(n)
while(frontier not empty)
  n = pull state from frontier
  if n = goal, return solution
  for all actions in n
    n' = a(n)
    if ((n' not visited or
        (visited and n'.cost < n_old.cost))
        priority = heuristic(goal,n') + cost
        frontier.append(priority)
        visited.append(n')
```

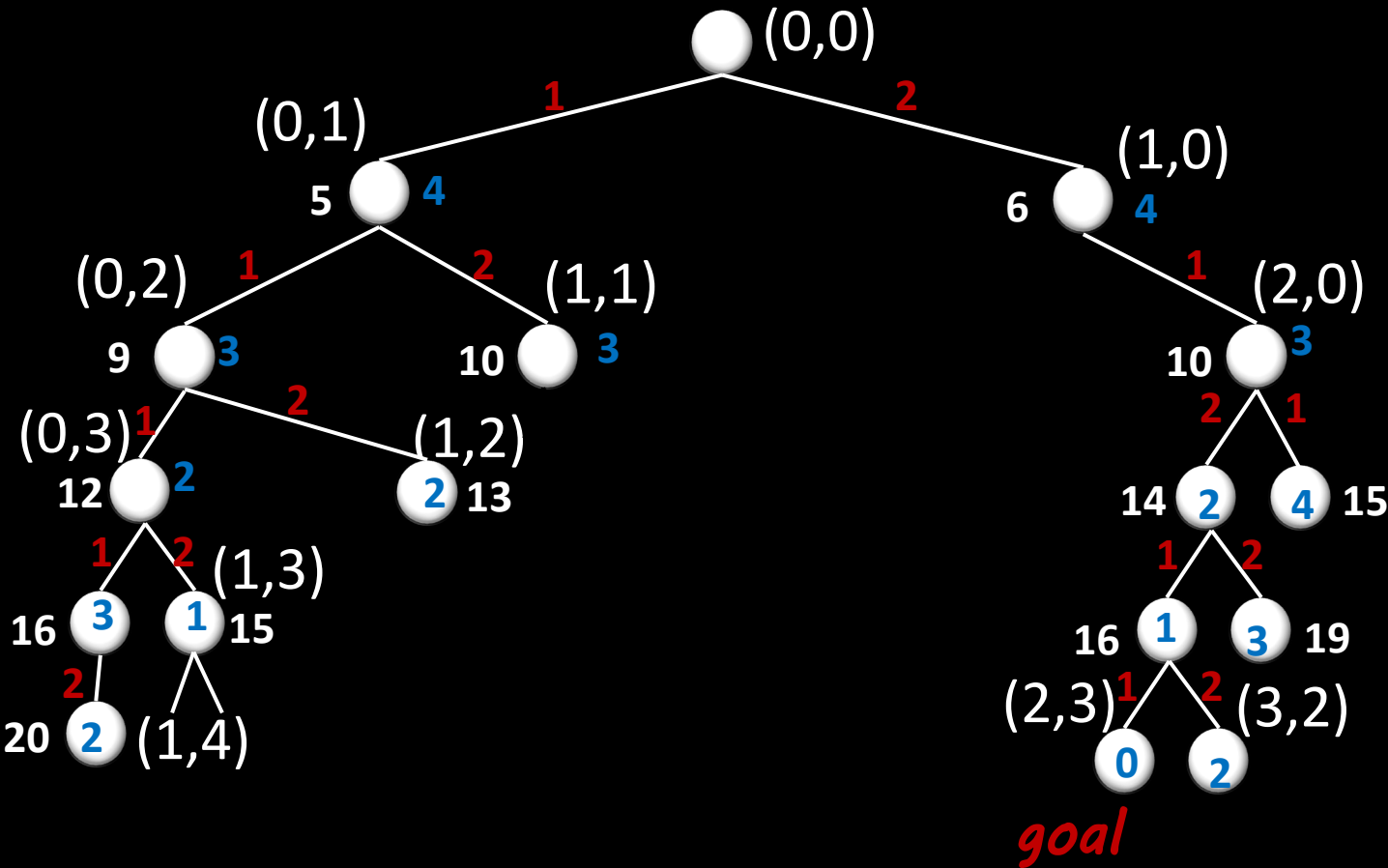
Search order: N, E, S, W

Find a treasure



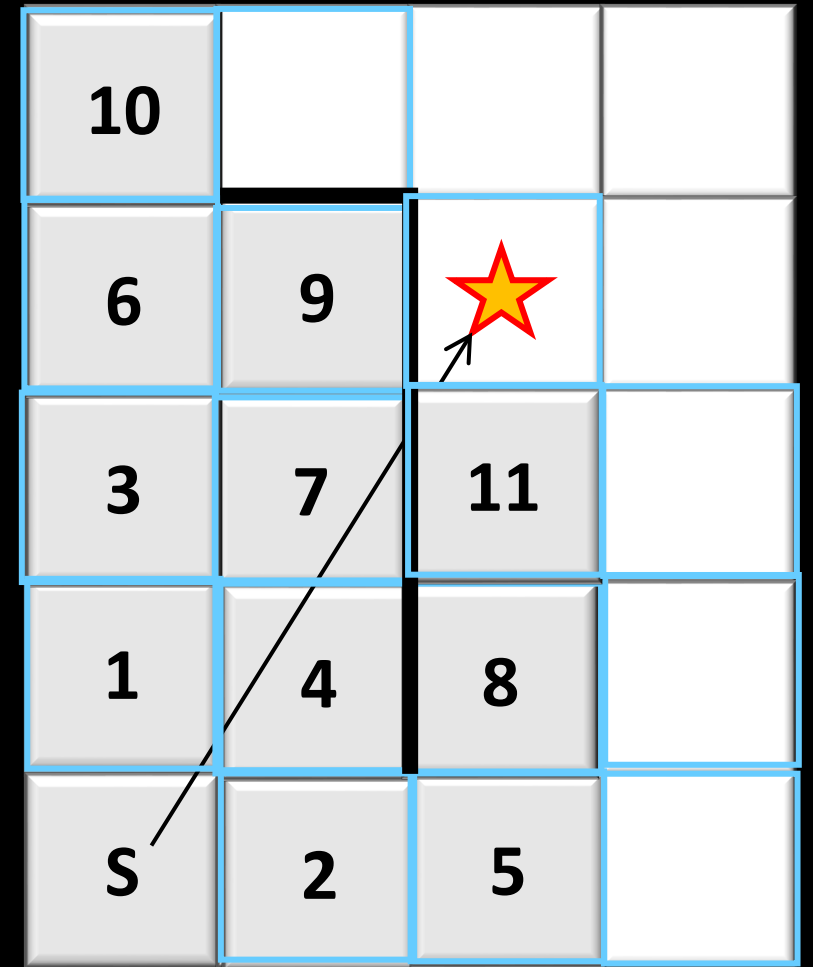
# Informed Search

- A\* ("A-star")
  - Cost and goal heuristic



Search order: N, E, S, W

Find a goal



y ↑

x →

# A\* Search

- What if the heuristic is too optimistic?
  - Estimated cost < true cost
- What if the heuristic is too pessimistic?
  - Estimated cost > true cost
  - No longer guaranteed to be optimal
- What if the heuristic is just right?
  - Pre-compute the cost between all nodes
  - Feasible for you?

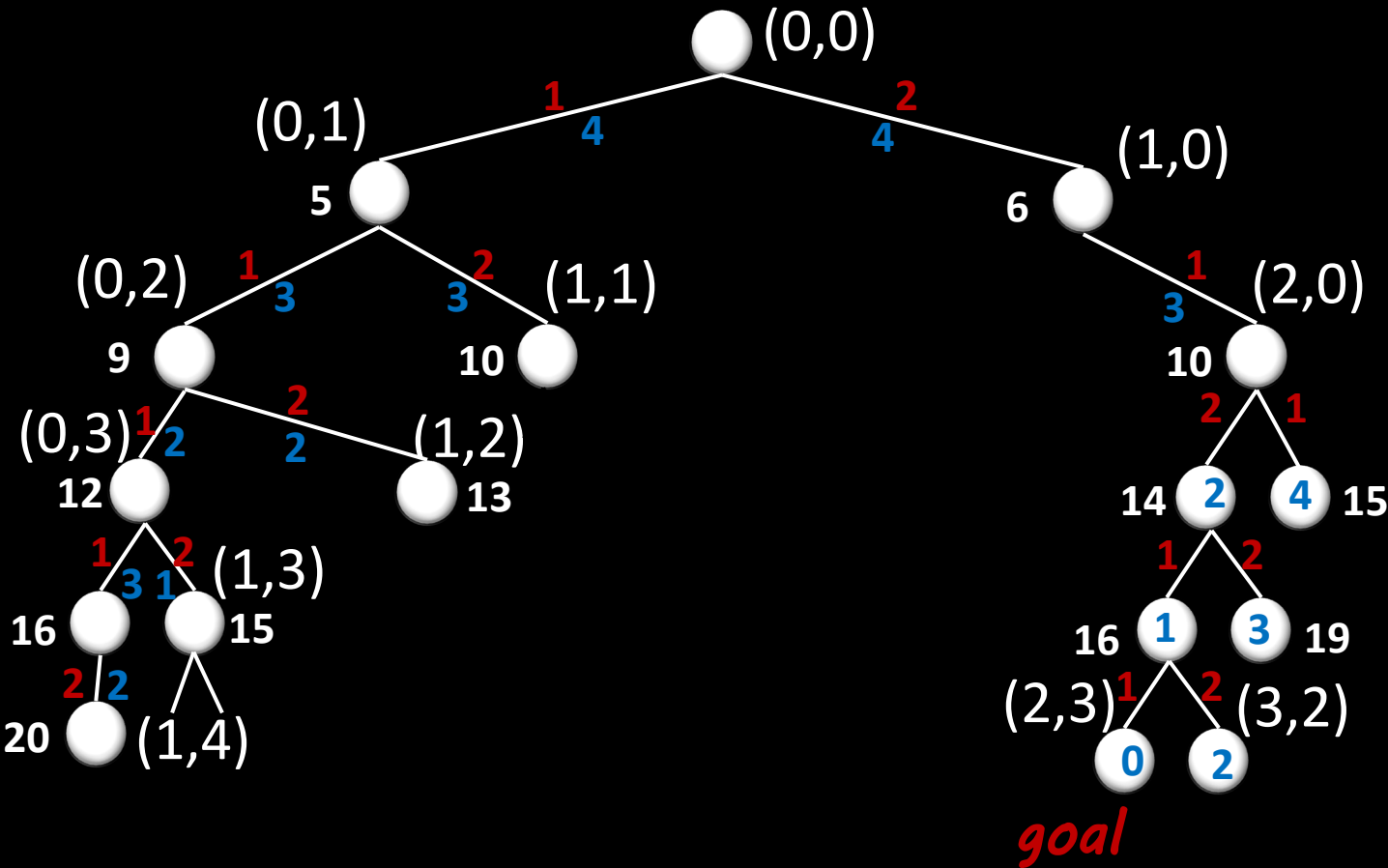
*admissible heuristic*

*inadmissible heuristic*



# Informed Search

- A\* (“A-star”)
  - Cost and goal heuristic



- Complete?
  - Yes!
- Time Complexity
  - $O(b^m)$
- Space Complexity
  - $O(b^m)$
- Optimal?
  - Yes, if the heuristic is admissible!

# Summary

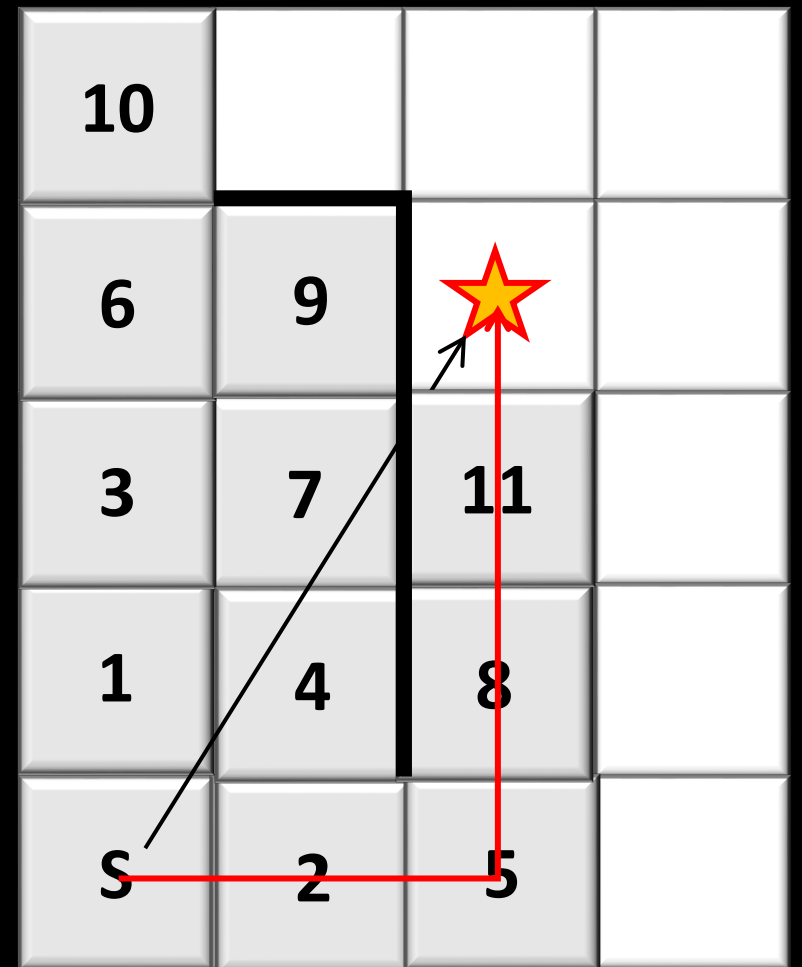
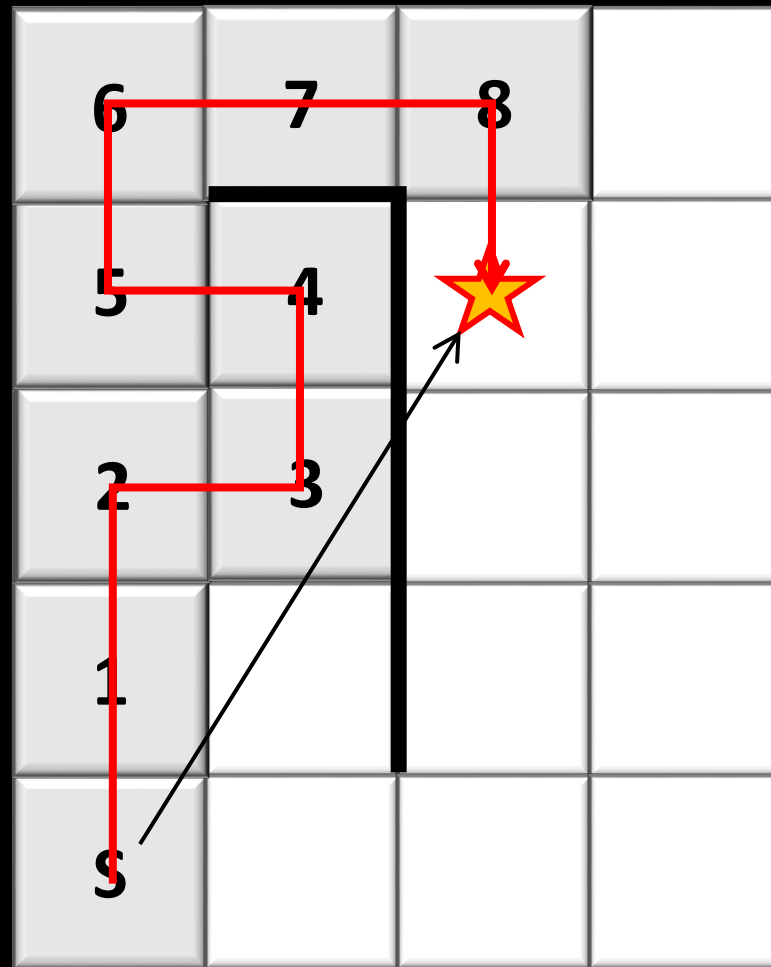
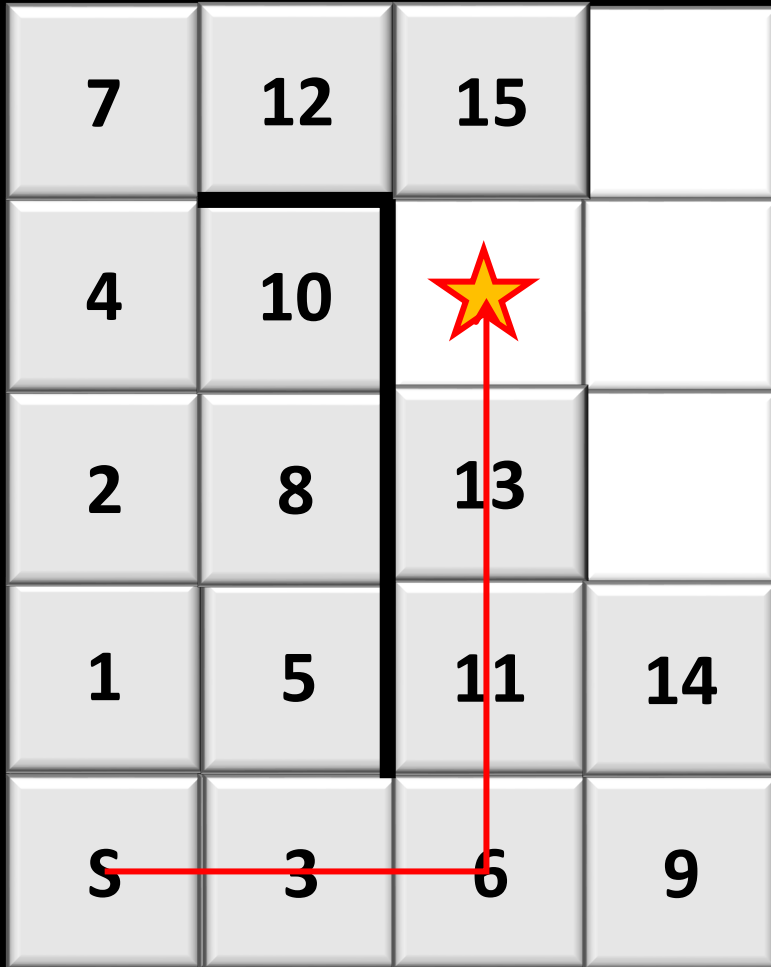
LCFS

*minimum path*

Greedy

A\*

*minimum path  
and efficient*



Upson Hall, 124 Hoy Rd, Ithaca, NY 14850  
Cornell Dairy Bar, 411 Tower Rd, Ithaca, NY 14850  
Add destination

OPTIONS

- Send directions to your phone
- via Hoy Rd  
18 min  
0.8 mile  
DETAILS
- via Hoy Rd and NY-366 E/Dryden Rd  
21 min  
1.0 mile
- via Hoy Rd and Tower Rd  
22 min  
1.0 mile

