

**ECE 4160/5160**  
**MAE 4910/5910**

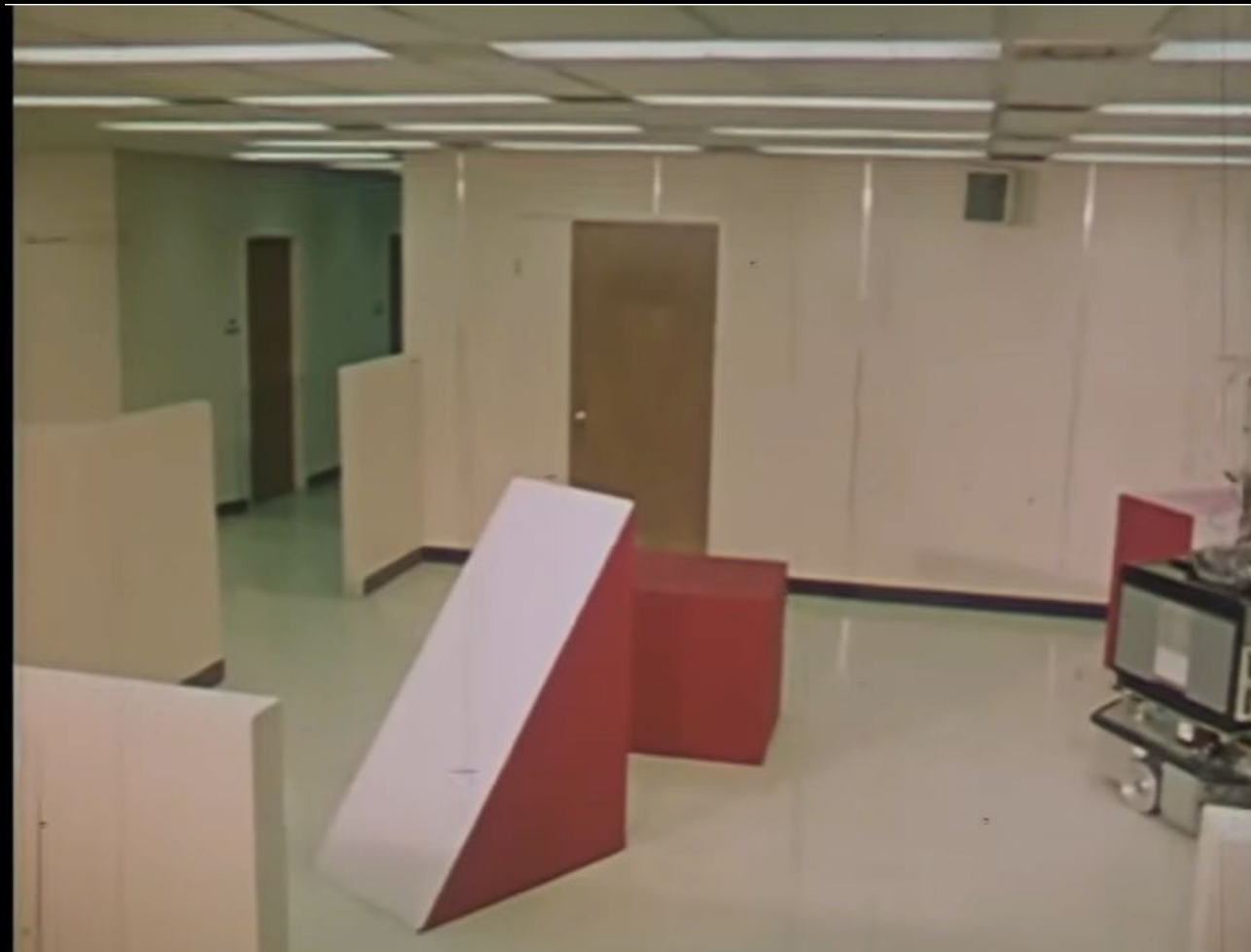
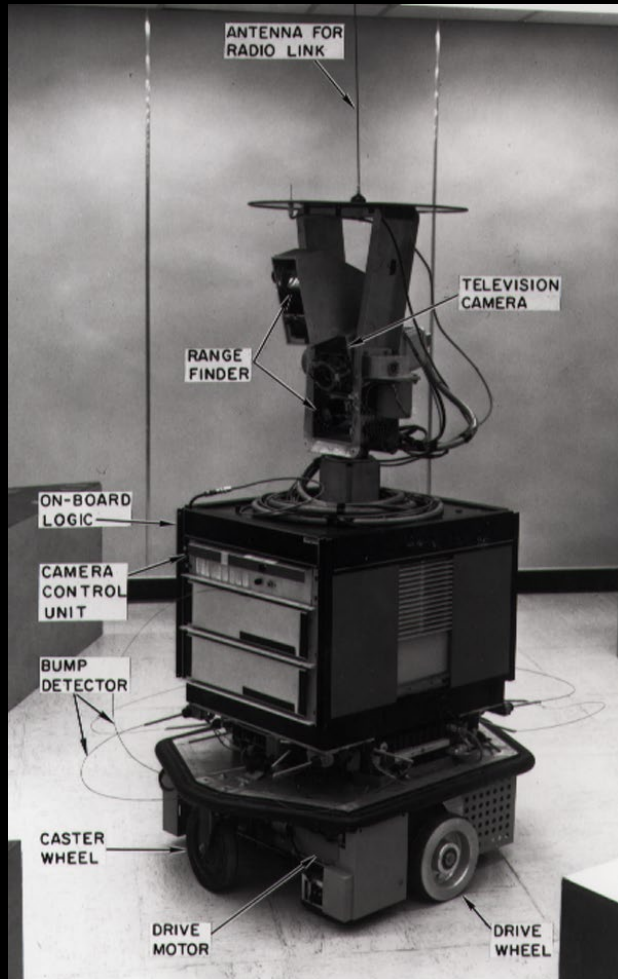
Prof. Kirstin Hagelskjær Petersen  
kirstin@cornell.edu

# Fast Robots

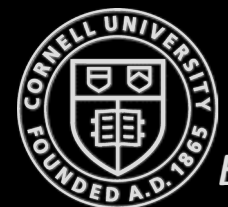
# Sensors

- Intro to sensors
- Distance sensors
- Odometry and dead reckoning
- Lab 2

# History

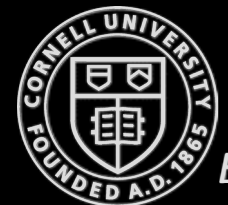


Shakey: Experiments in Robot Planning and Learning (1972), SRI



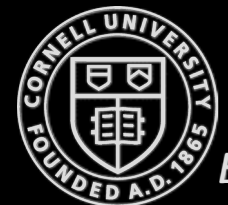
# Sensor Classification

- **Proprioceptive**
  - Motor speed, wheel load, joint angles, battery voltage
- **Exteroceptive**
  - distance measurements, light intensity, sound amplitude
- **Passive Sensors**
  - Measure ambient environmental energy
  - E.g. temperature probes, microphones, light sensors
- **Active Sensors**
  - Senses reaction to emitted energy
  - E.g. wheel quadrature encoders, ultrasonic sensors, laser rangefinders



# Classification

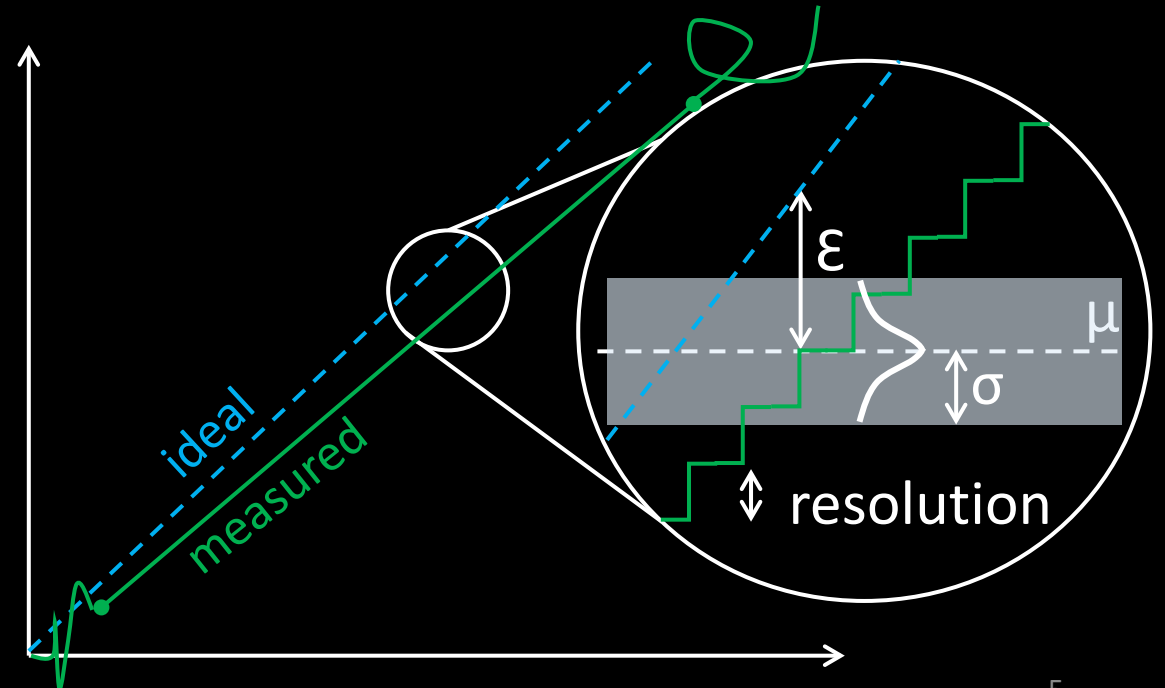
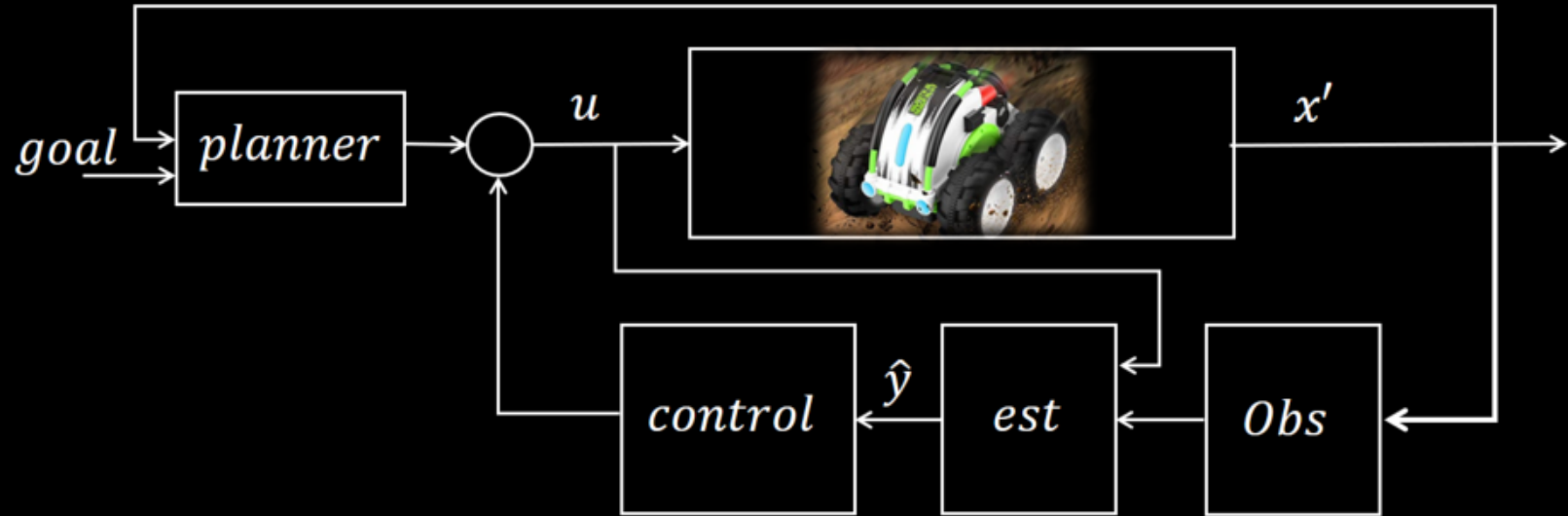
| Type                           | Sensor   | Prop/Exte  | Passive/Active                         |
|--------------------------------|--|--|--|
| Tactile<br>(contact/closeness) | Contact switches, bumpers,<br>Break beams, proximity<br>Capacitive                             | Exteroceptive<br>Exteroceptive<br>Exteroceptive                      | Passive<br>Active<br>Both              |
| Wheel/motor                    | Brush encoders<br>Potentiometers<br>Optical encoders<br>Magnetic/inductive/capacitive encoders | Proprioceptive<br>Proprioceptive<br>Proprioceptive<br>Proprioceptive | Passive<br>Passive<br>Active<br>Active |
| Active ranging                 | Reflectivity sensors, ultrasonic, laser<br>rangefinders, optical triangulation, etc.           | Exteroceptive  | Active                                 |
| Heading                        | Compass<br>Gyroscopes  | Exteroceptive<br>Proprioceptive                                      | Passive<br>Passive                     |
| Ground based beacons           | GPS, RF, reflective beacons  | Exteroceptive  | Active                                 |
| Motion/speed                   | Doppler radar, sound   | Exteroceptive  | Active                                 |
| Vision                         | CCD/CMOS   | Exteroceptive  | Passive                                |



# Sensor Characteristics

*Name some examples*

- Dynamic Range
- Range
- Resolution
- Linearity
- Bandwidth / Sampling Frequency
- Sensitivity
- Cross-sensitivity
- Accuracy
- Precision
- Error
  - Systematic
  - Random
- Power consumption
- Size, price, etc...



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# Distance Sensors

# DIY-level Distance Sensors

| Technology         | Application | Pros   | Cons  |
|--------------------|-------------|--|---|
| Amplitude-based IR | <10cm       | <ul style="list-style-type: none"> <li>• ~ 0.5 USD</li> <li>• Small form factor</li> </ul>   | <ul style="list-style-type: none"> <li>• Depends on target reflectivity</li> <li>• Does not work in high ambient light</li> </ul>   |
| IR triangulation   | <1m         | <ul style="list-style-type: none"> <li>• Insensitive to surface color/texture/ambient light</li> </ul>   | <ul style="list-style-type: none"> <li>• ~ 10 USD</li> <li>• Does not work in high ambient light</li> <li>• Bulky (1.75" × 0.75" × 0.53")</li> <li>• Low sample rate (26Hz)</li> </ul>  |
| IR Time of Flight  | 0.1 - 4m    | <ul style="list-style-type: none"> <li>• High sample rate (4kHz)</li> <li>• Small form factor</li> <li>• Insensitive to surface color/texture/ambient light</li> </ul> | <ul style="list-style-type: none"> <li>• ~ 6.5 USD</li> <li>• Complicated processing</li> <li>• Low sampling frequency: 7-30Hz</li> </ul>   |
| Ultrasonic         | 0.2 – 10m   | <ul style="list-style-type: none"> <li>• Low cost</li> <li>• Insensitive to ambient light and surface color</li> <li>• Works in rain and fog</li> </ul>                | <ul style="list-style-type: none"> <li>• ~4 USD</li> <li>• Complicated processing</li> <li>• Resolution trade off with max range</li> <li>• Output depends on surface/geometry/humidity</li> <li>• Bulky, sample time (tens of milliseconds)</li> <li>• Hard to achieve a narrow FoV</li> </ul> |

# The Electromagnetic Spectrum



Radio

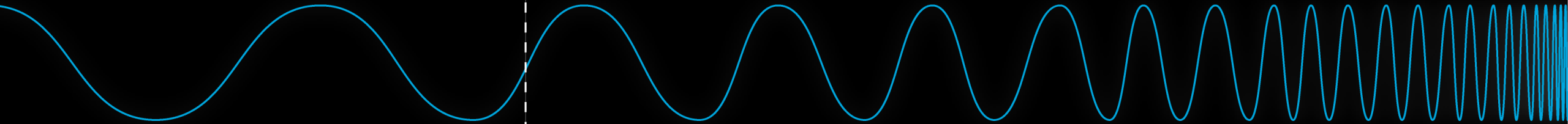
Infrared

Visible

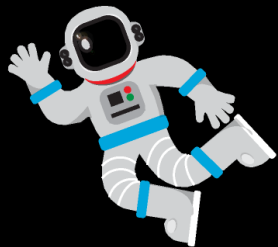
Ultraviolet

X-Ray

Gamma Ray



NASA HQ



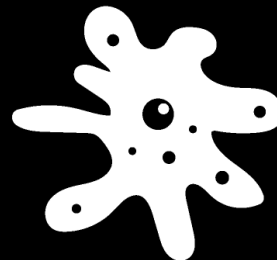
Astronaut



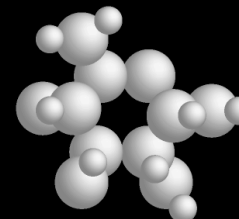
Coin



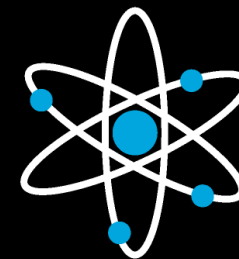
Pinhead



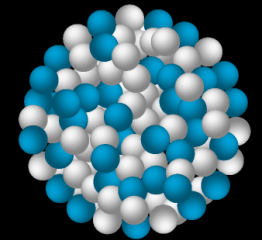
Amoeba



Molecule



Atom



Atomic Nuclei

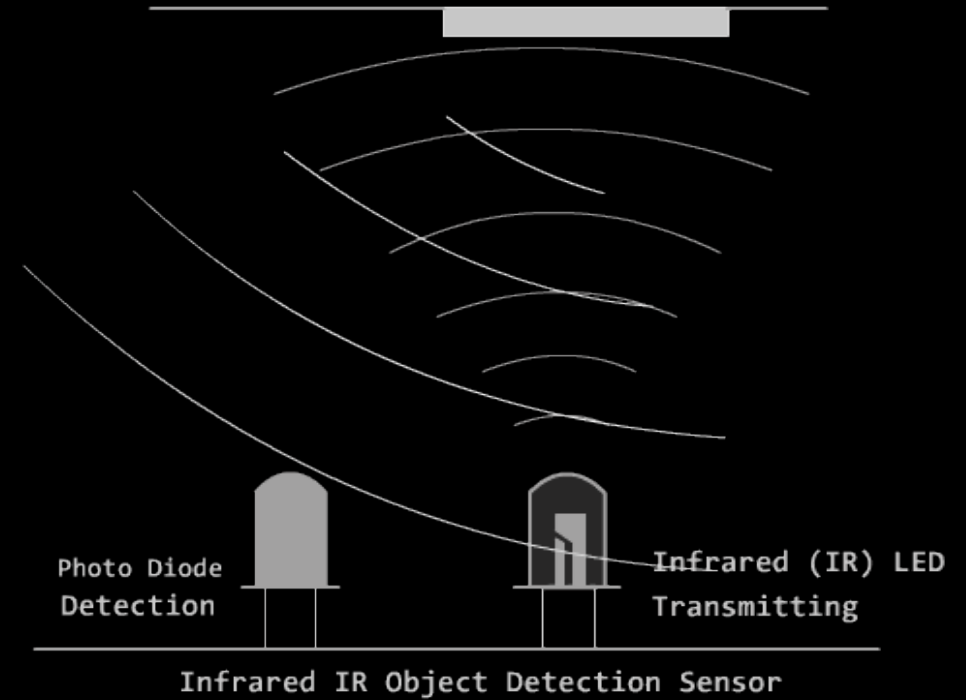


# Amplitude –Based IR Distance Sensors

- Very cheap
- Very simple circuitry
- Works reasonably well for
  - Object detection
  - Break beam sensors
  - Classifying greyscale intensity at a fixed distance
  - Short-range distance sensor
- Range <0.5m
- Sensitive to surface color, texture, and ambient light

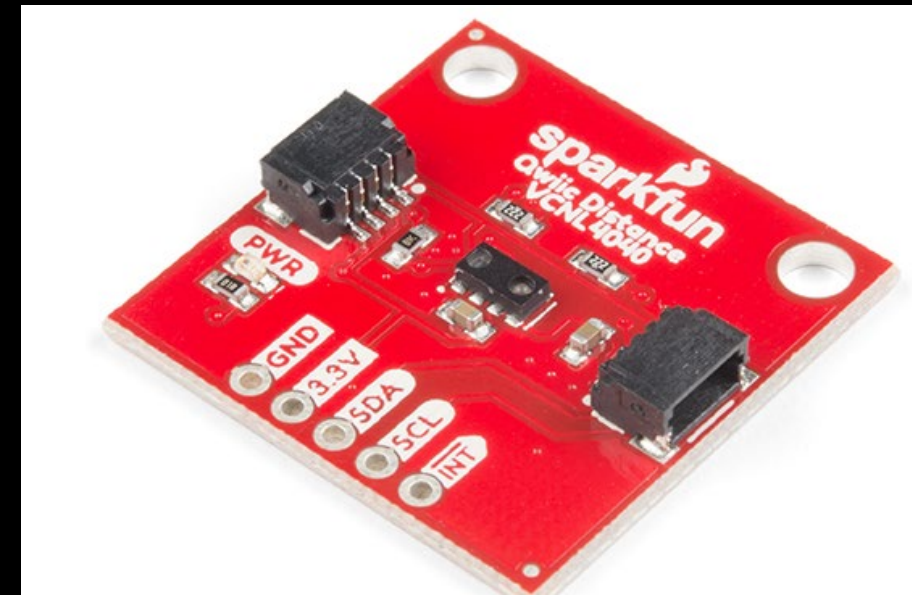
www.rakeshmondal.info

Point of Impact



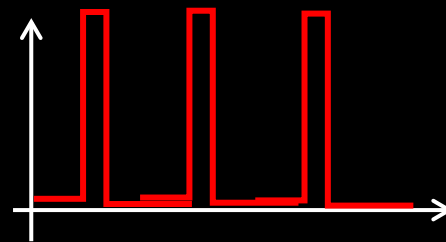
## VCNL4040

- \$3.34
- Range 20cm
- Ambient light sensor
- Programmable DC



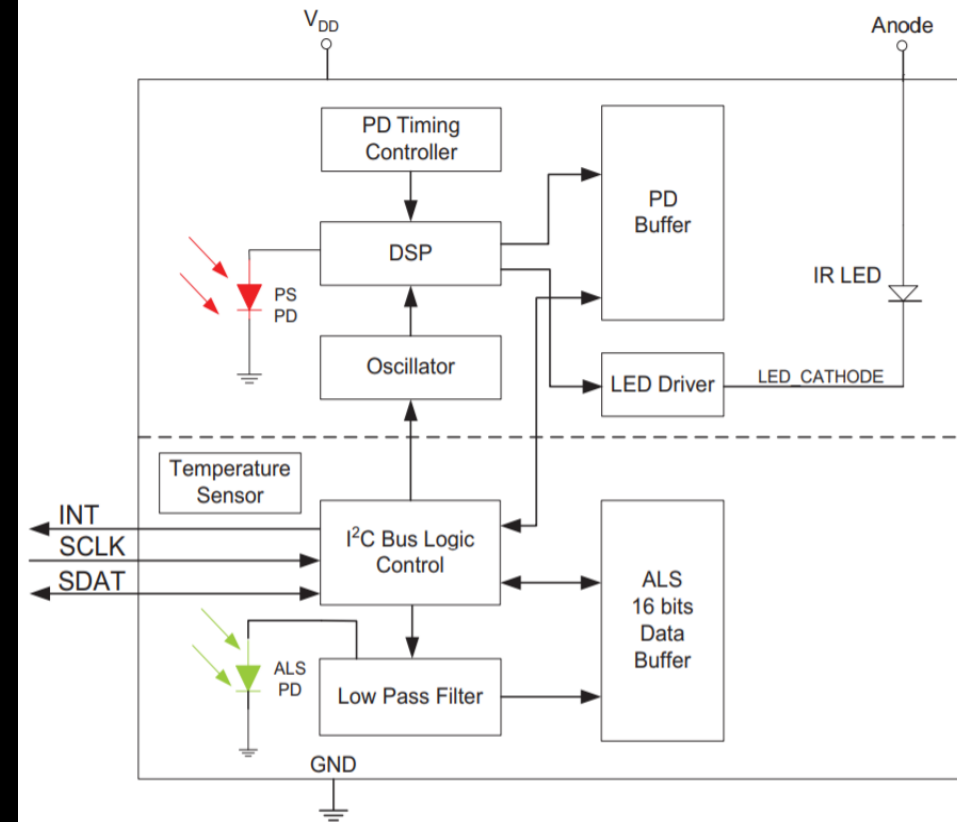
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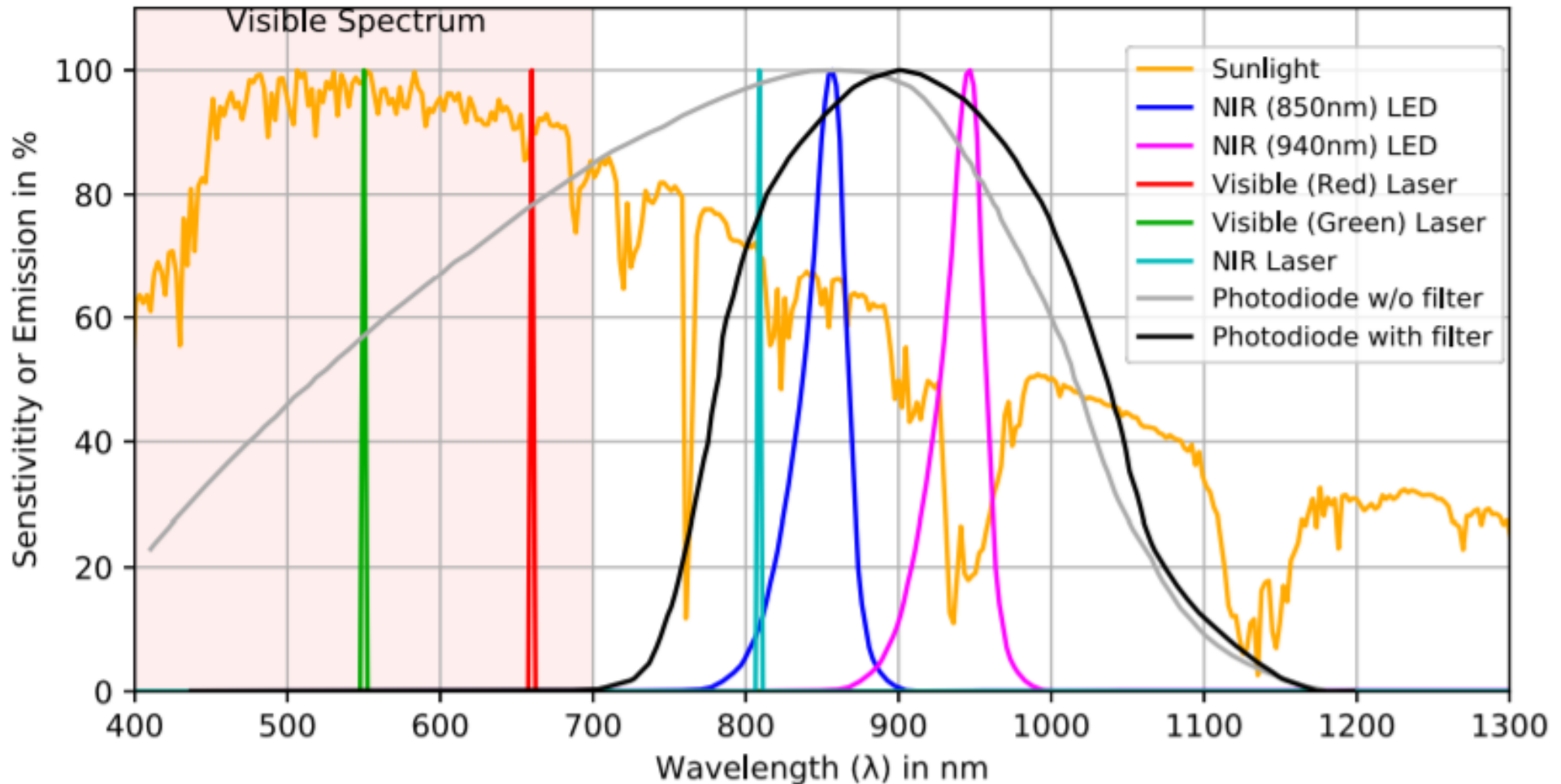
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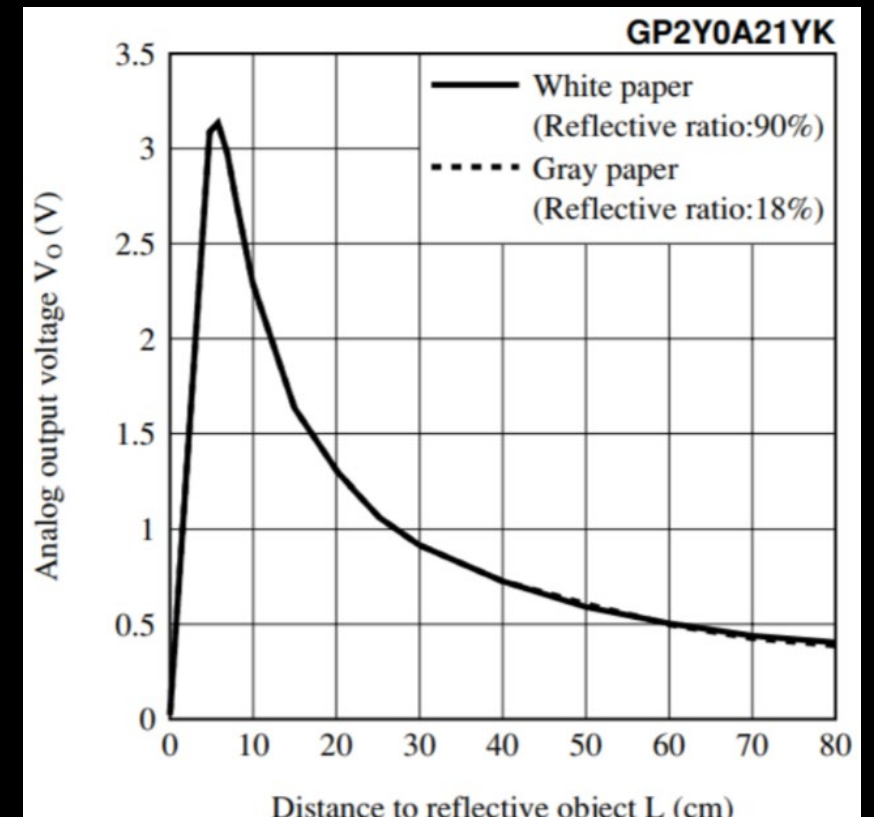
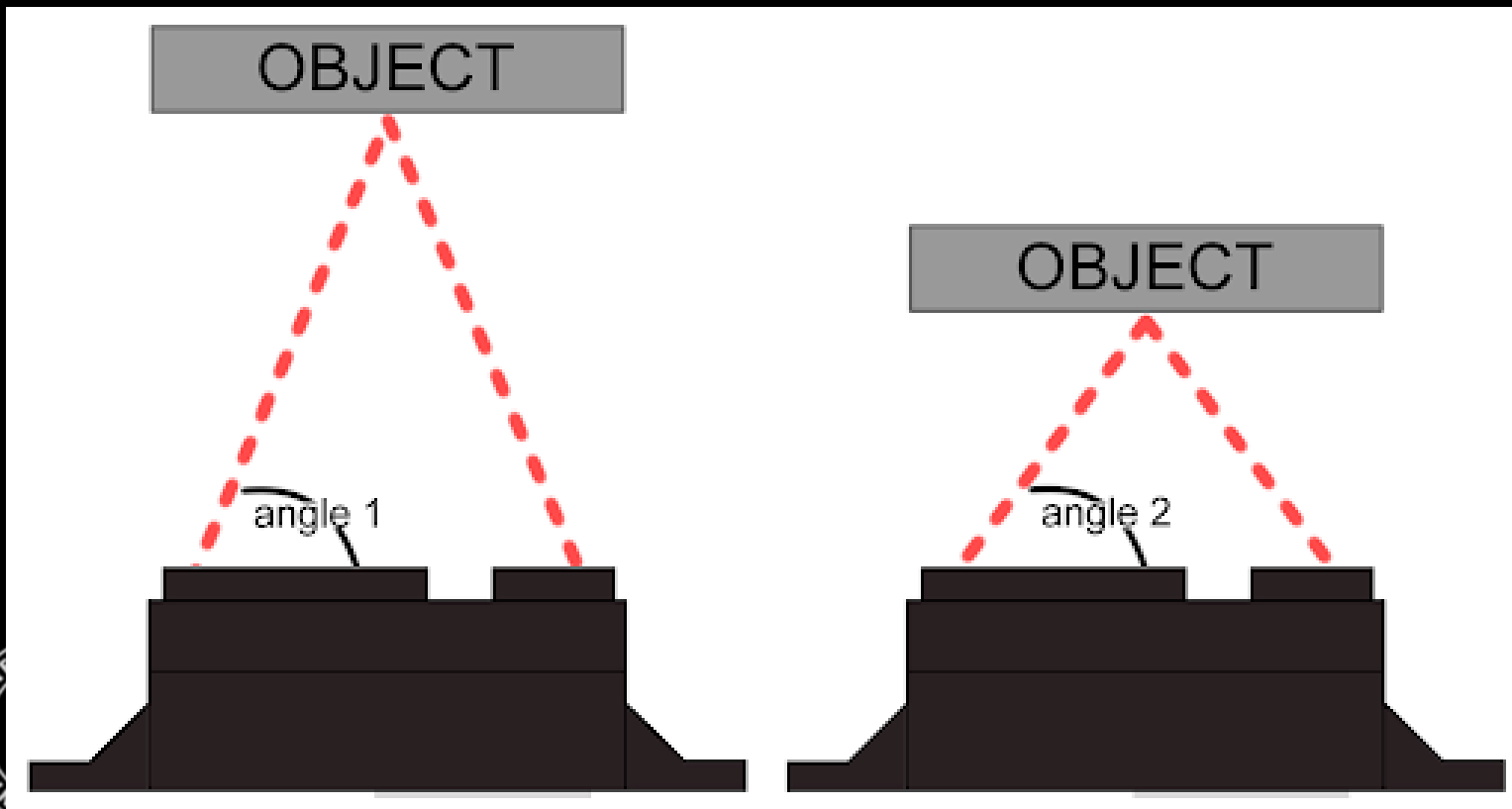
Normalized Spectral Sensitivity(Photodiodes)/Emission(Emitters) for components



# Triangulation-Based IR Distance Sensors

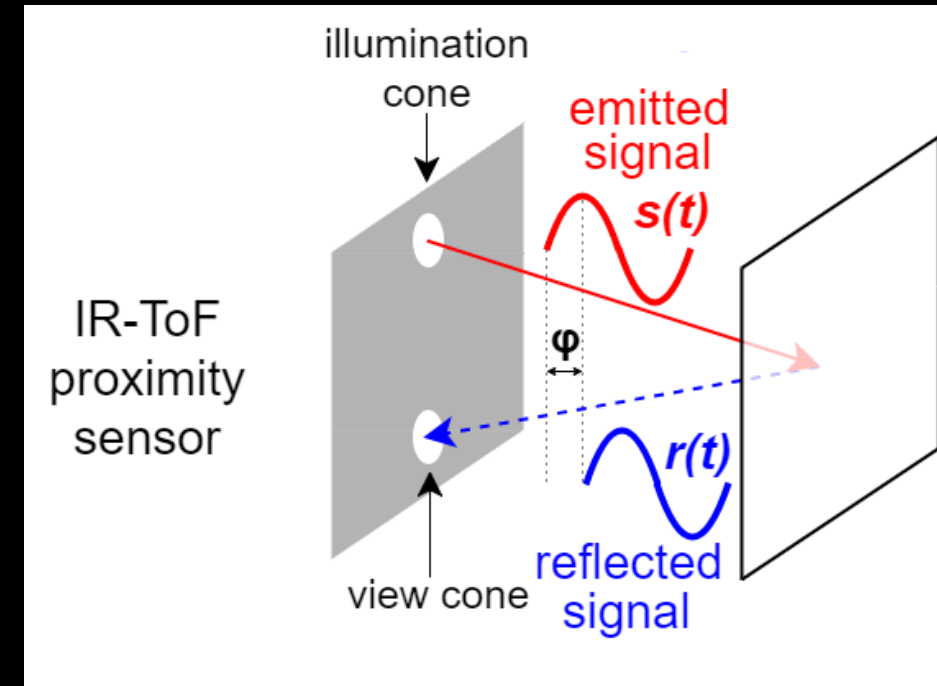
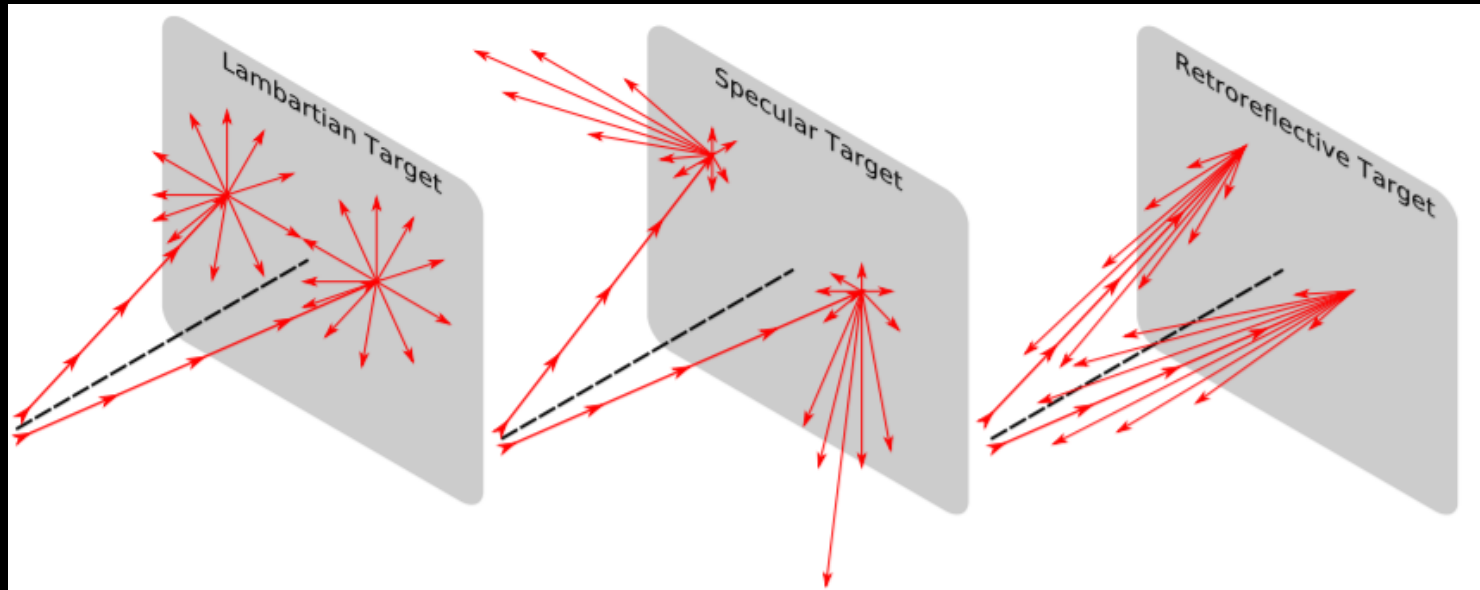


- Very simple circuitry
- Less sensitive to color, texture, ambient light
- Medium range (0.05 - 1 m)
- Cost 5-25 USD



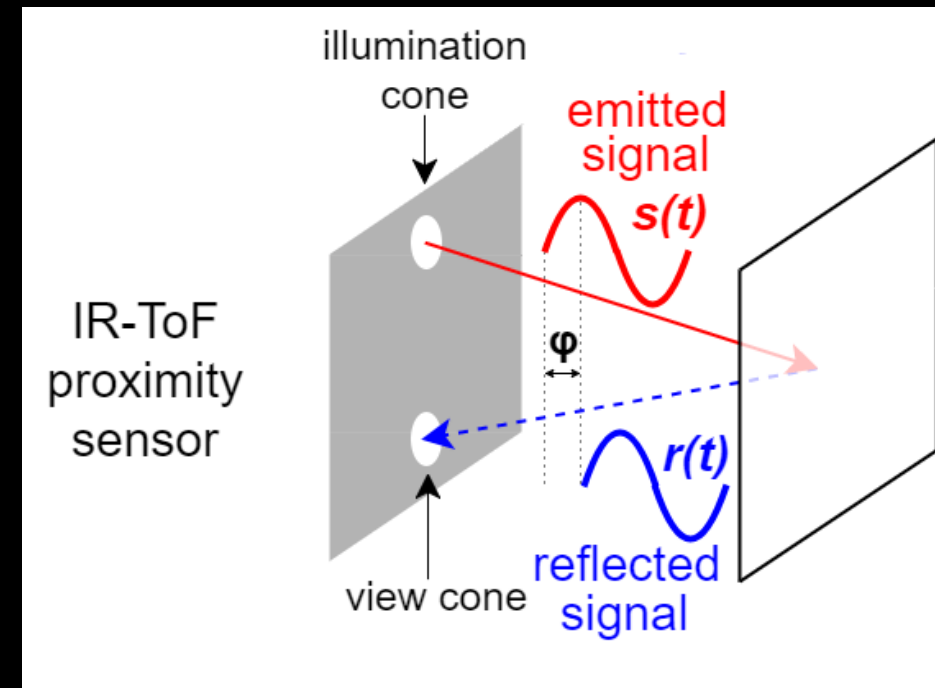
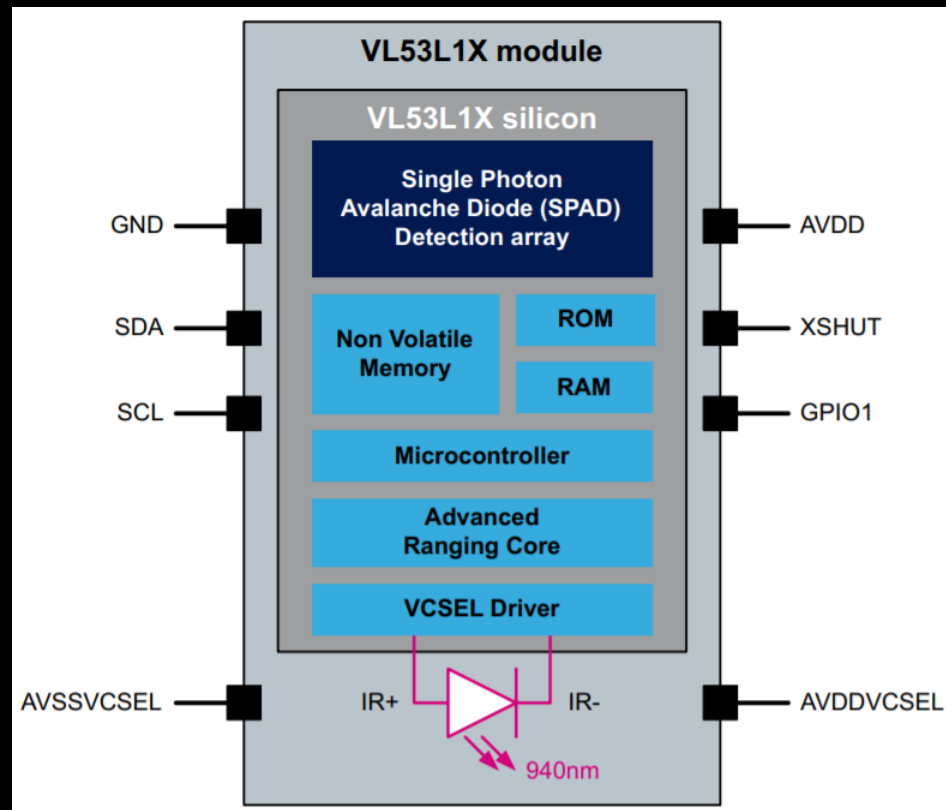
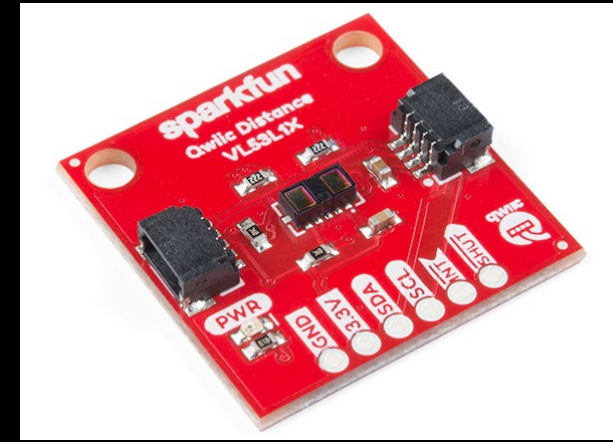
# Time of Flight IR Sensor

- Emit a pulse modulated signal, record time  $t$  until return!
  - $r = t * c / 2$
  - $c = \text{speed of light} = 299,792,458 \text{ m/s}$
- Mostly insensitive to texture, color, ambient light



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- Emit a pulse modulated signal, record time  $t$  until return!
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- Outputs (Distance in mm, return signal rate, ambient signal rate, range status)

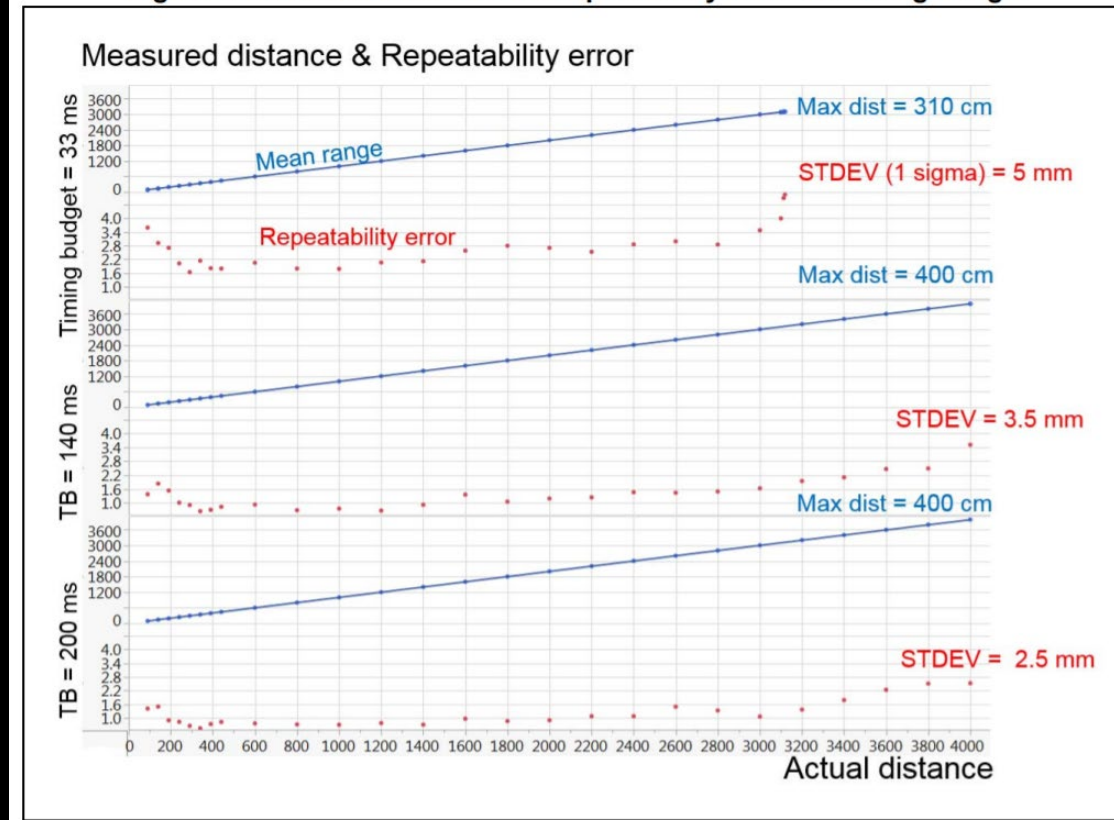


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- Mostly insensitive to texture, color, ambient light
- Outputs (Distance in mm, return signal rate, ambient signal rate, range status)
- Programmable FOV
- Distance mode (~1, 2, 4m)
- Timing budget
  - 20ms: short distance mode (0.05 - 1.3m)
  - 33ms: all distance modes (0.05 - 3.6m)
  - 140ms: improve reliability errors
- Newest developments
  - ToF Imager (64 pixels)

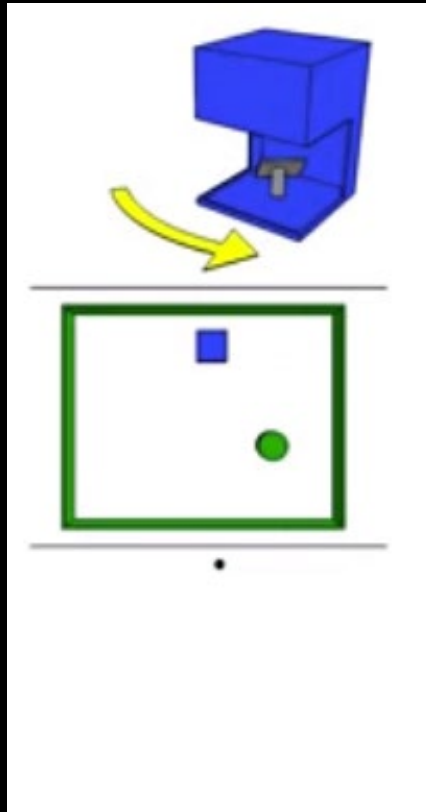


Figure 6. Maximum distance and repeatability error vs. timing budget

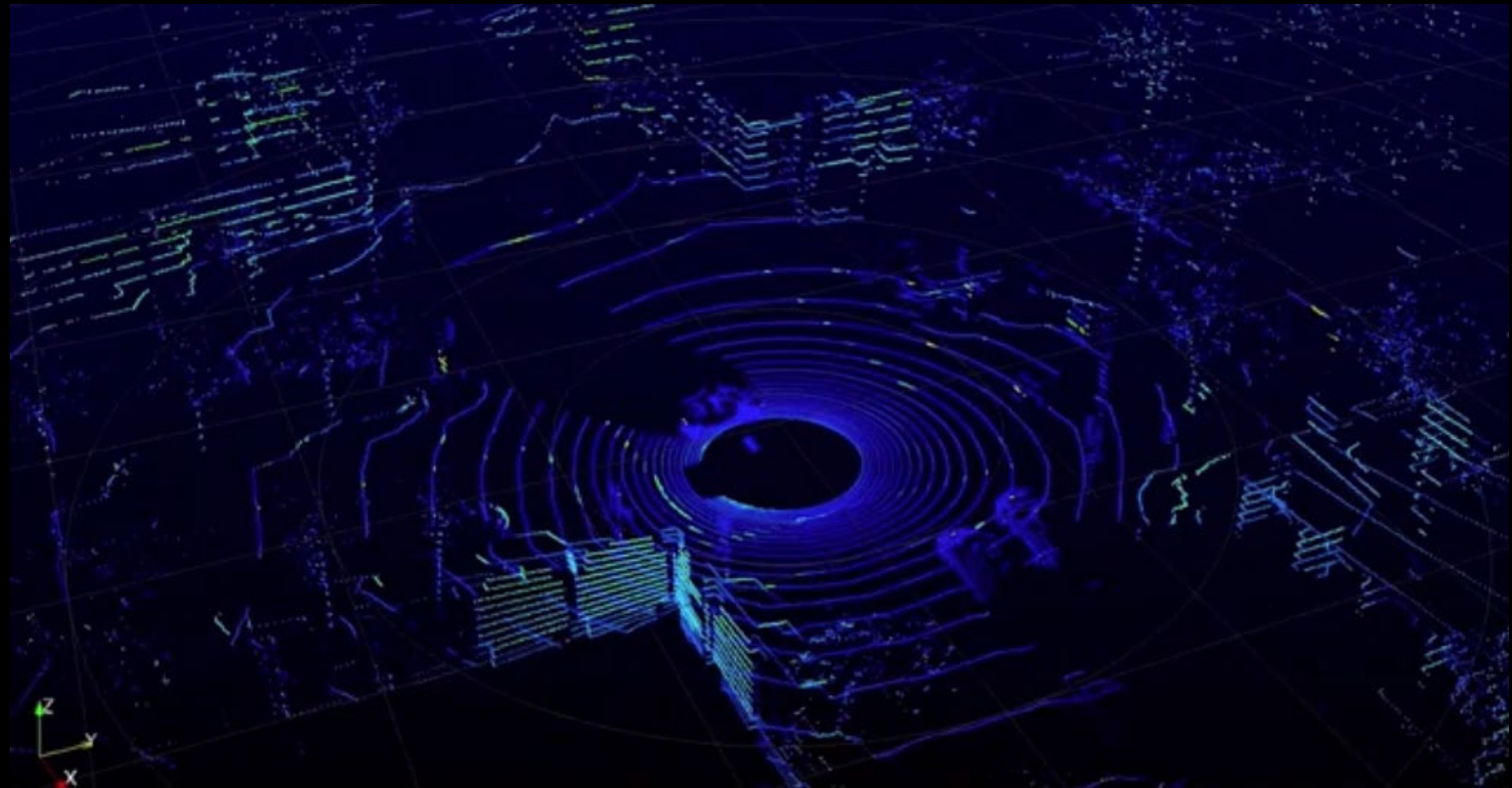


# Light Detection and Ranging Sensors

- Most common sensors on autonomous cars and robots
- Single points, line scans, full 3D
- \$\$\$



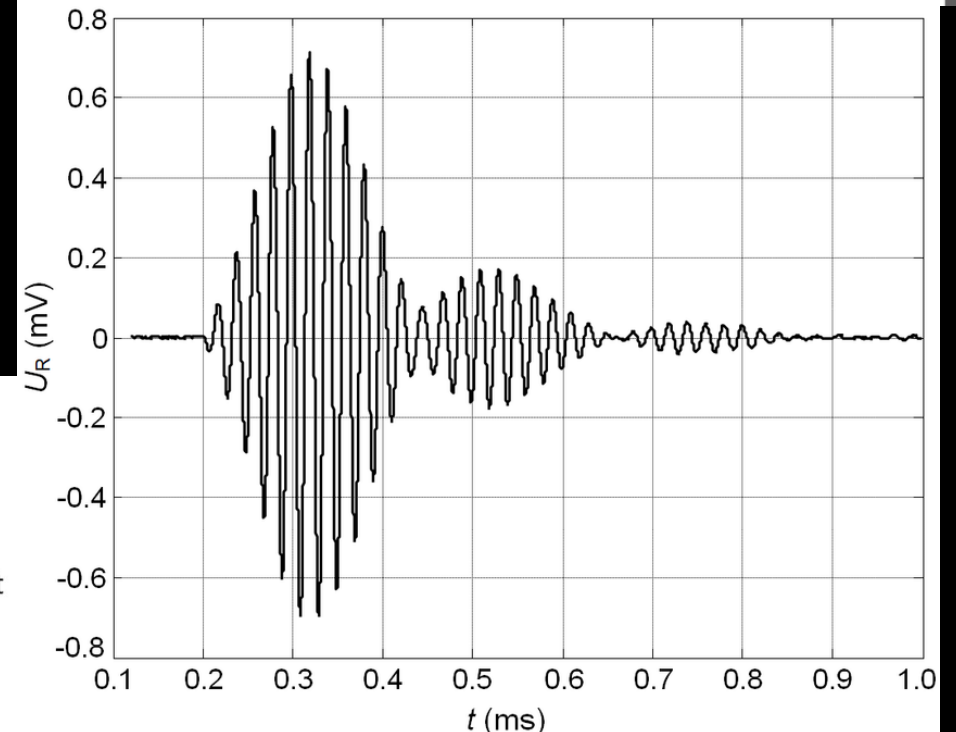
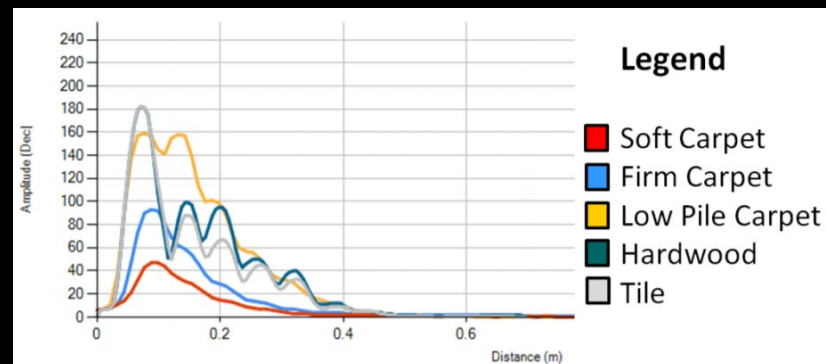
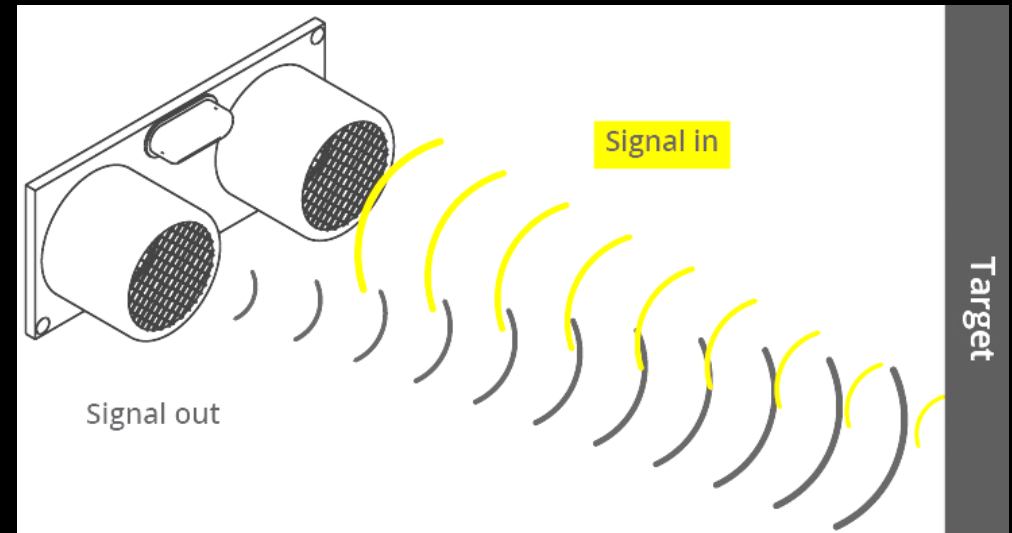
*What does the color represent?*





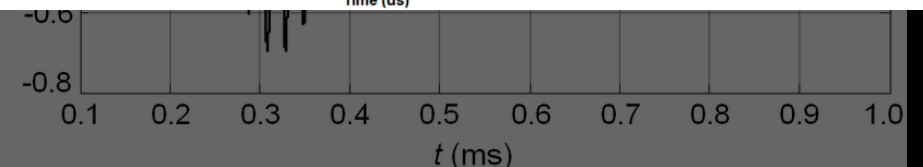
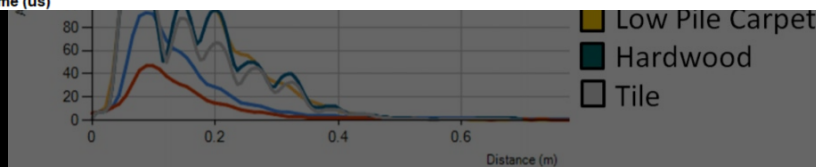
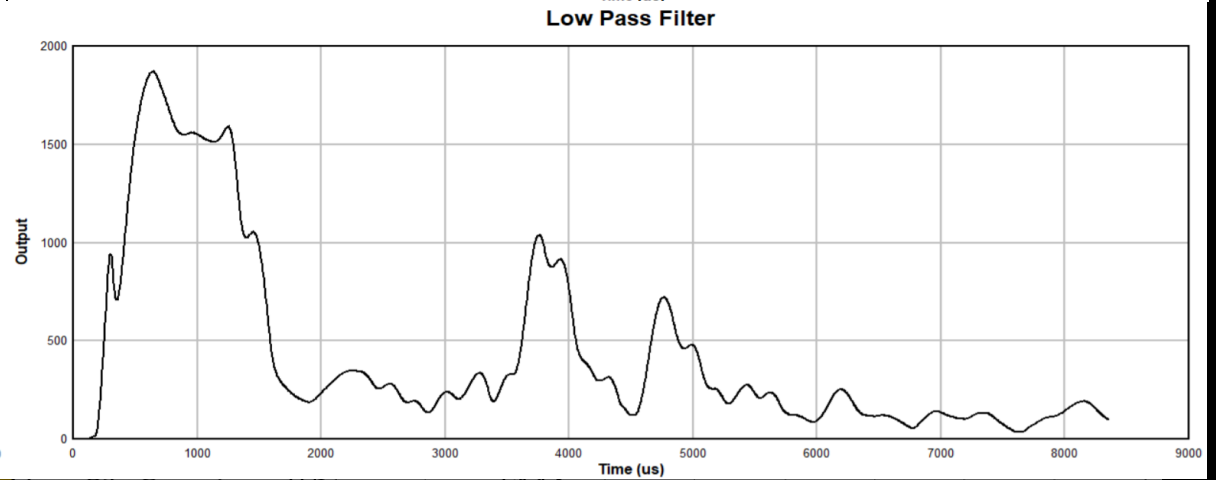
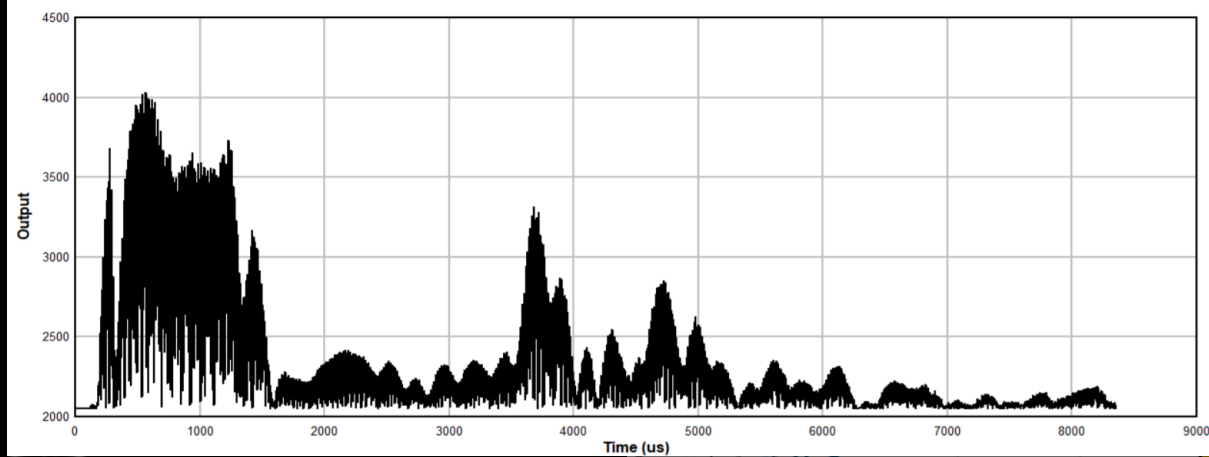
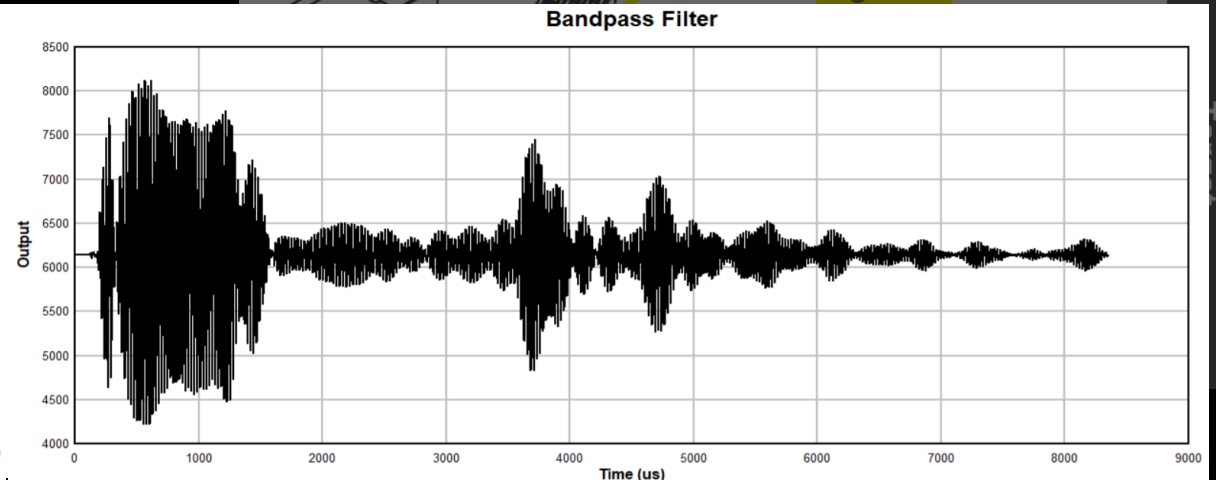
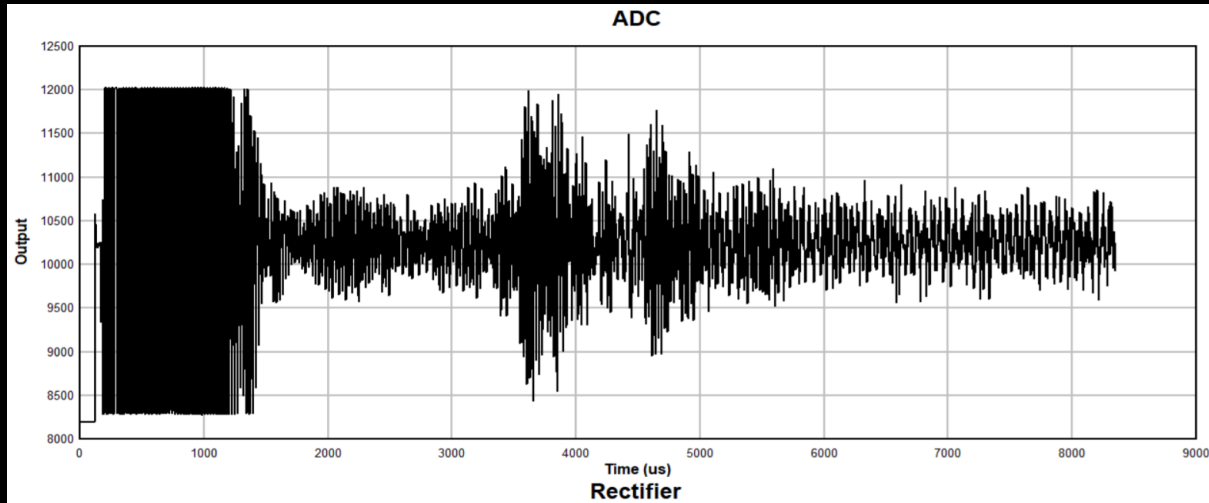
# Ultrasound (Time of Flight) Distance Sensors

- Measure the reflections of an emitted sound wave
  - $r = t * c_{\text{sound}}/2$
  - $c_{\text{sound}} = 343 \text{ m/s}$
- Frequency versus resolution and range
  - 58kHz: cm resolution, range < 11m
  - 300kHz: mm resolution, range < 0.3m
- Cost is low (Sparkfun module: 4-12 USD)
- Insensitive to color, texture, glass, fog, dust, etc.
- Sensitive to humidity, temperature, audible noise, and geometry



# Ultrasound (Time of Flight) Distance Sensors

- Measure the reflections of an emitted sound wave



# DIY-level Distance Sensors

| Technology         | Application | Pros  | Cons  |
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# ODOMETRY SENSORS

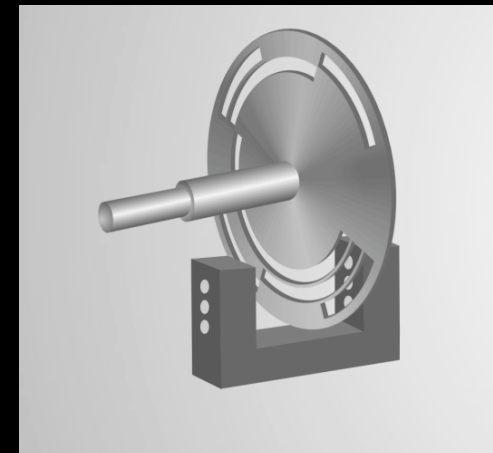
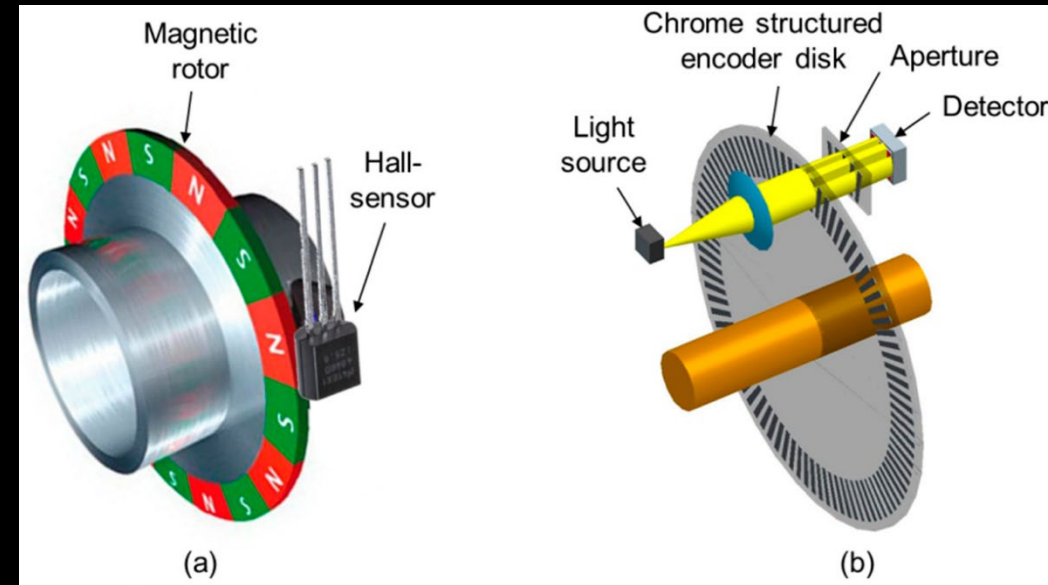
(the process of inferring your position by the integration of speed)

- Wheel encoders
  - IMU
- Optical flow

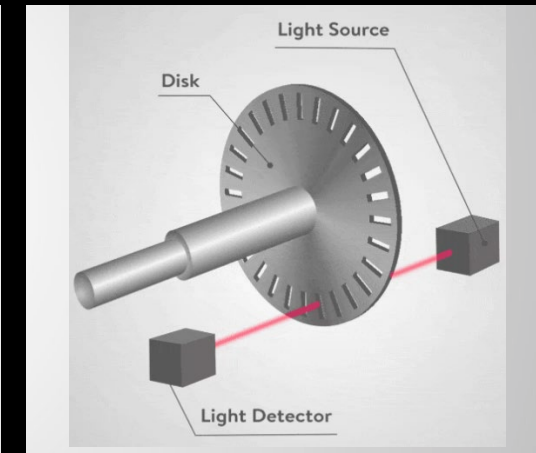
# Encoders

- Technology
  - Magnetic
  - Optical
  - Inductive, Capacitive, Laser
- Rotary (shaft) Encoders
  - Absolute Rotary Encoders (angular position)
  - Incremental Rotary Encoders (distance, speed, position)

*How to add encoders to your robot?*



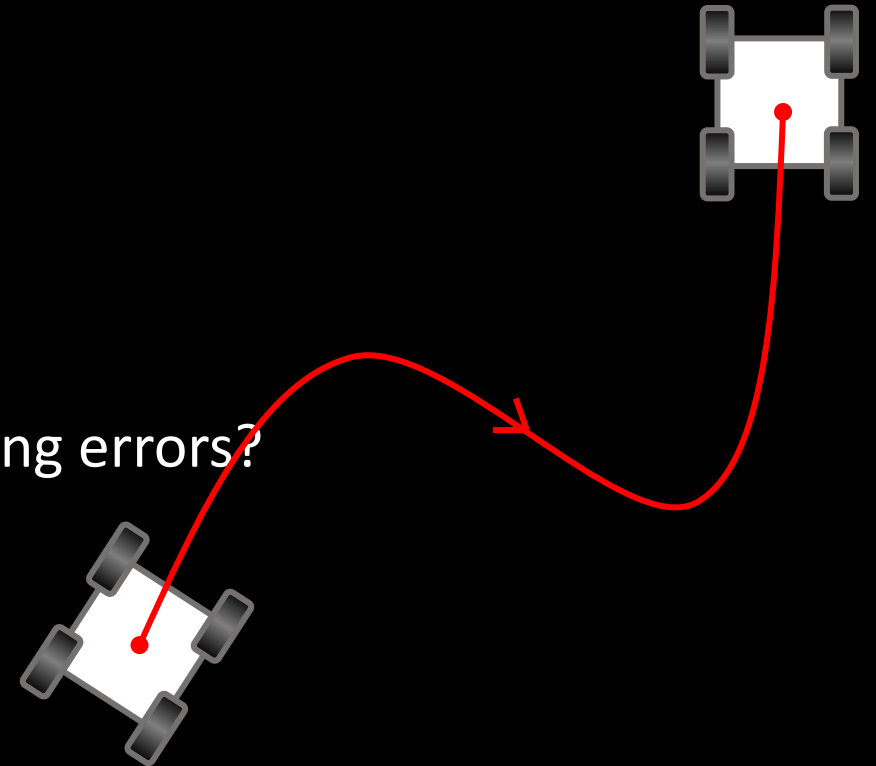
Absolute Rotary Encoder



Incremental Rotary Encoder

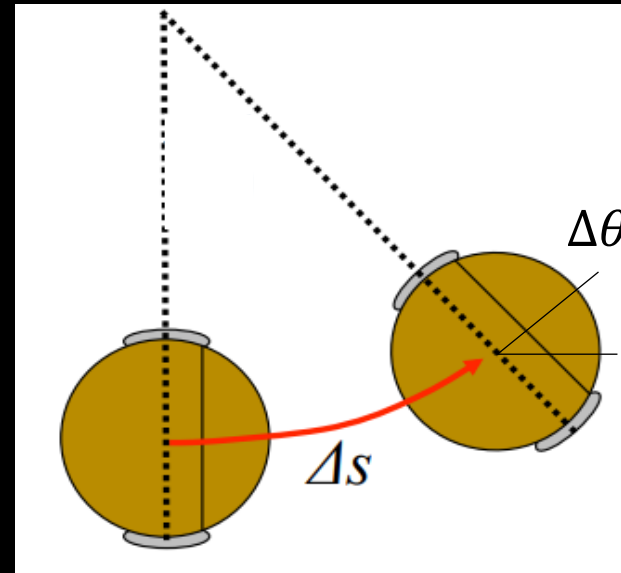
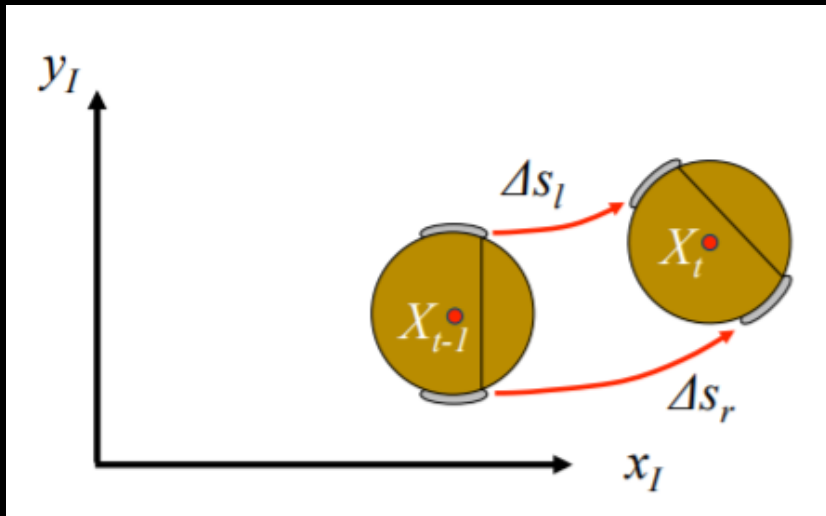
# Dead Reckoning

- Map the present state and wheel encoder measurements to the new robot state
  - $X_t = f(X_{t-1}, U_{t-1})$
  - Pro: Easy to implement
  - Con: Errors integrate and grow unbounded
- **Sources of error?**
  - Limited resolution during integration
  - Unequal wheel diameter
  - Variation in the contact point of the wheel
  - Variable friction > slipping
  - Drift or noise in sensors
- How do wheel rotation errors propagate into positioning errors?



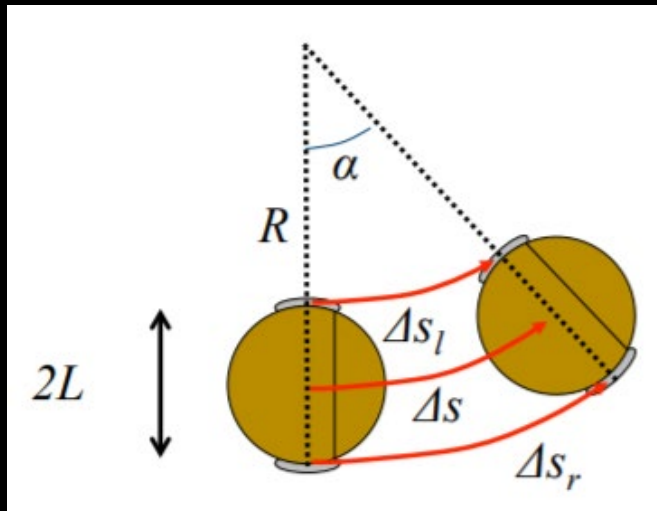
# Modeling Motion

- Start at pose  $X_{t-1}$ , move right/left wheel by  $\Delta s_r$  and  $\Delta s_l$ , what is pose  $X_t$ ?
- Model the change in angle  $\Delta\theta$  and the distance travelled  $\Delta s$ 
  - (assume that the robot is travelling on a circular arc of constant radius)



# Modeling Motion

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For circular arcs:

- (1)  $\Delta s_l = R\alpha$
- (2)  $\Delta s_r = (R + 2L)\alpha$
- (3)  $\Delta s = (R + L)\alpha$

- Use (1) and (2) to compute (4):

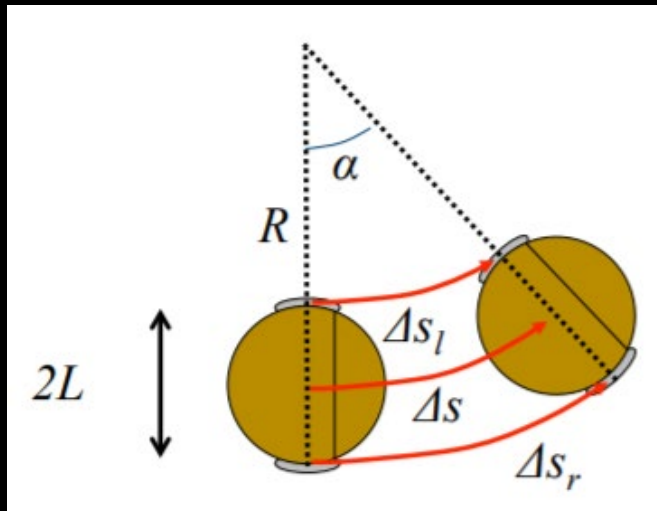
- $$L\alpha = \frac{(\Delta s_r - R\alpha)}{2}$$
- $$= \frac{\Delta s_r}{2} - \frac{\Delta s_l}{2}$$

- Insert into (3): 
$$\Delta s = \Delta s_l + \frac{\Delta s_r}{2} - \frac{\Delta s_l}{2} = \frac{\Delta s_l + \Delta s_r}{2}$$



# Modeling Motion

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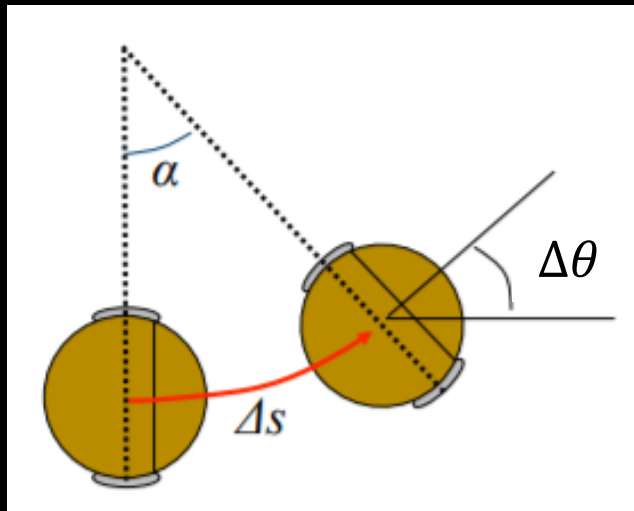


For circular arcs:

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  - (3)  $\Delta s = (R + L)\alpha$
  - (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- (or note that the distance traveled by the robot center, is simply the avg distance traveled by each wheel)

# Modeling Motion

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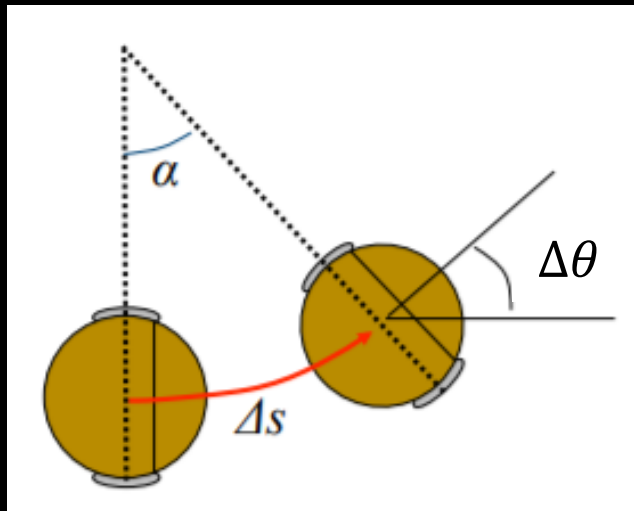


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- (3)  $\Delta s = (R + L)\alpha$
- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- The change in angle,  $\Delta\theta$ :
  - $\Delta\theta = \alpha$

# Modeling Motion

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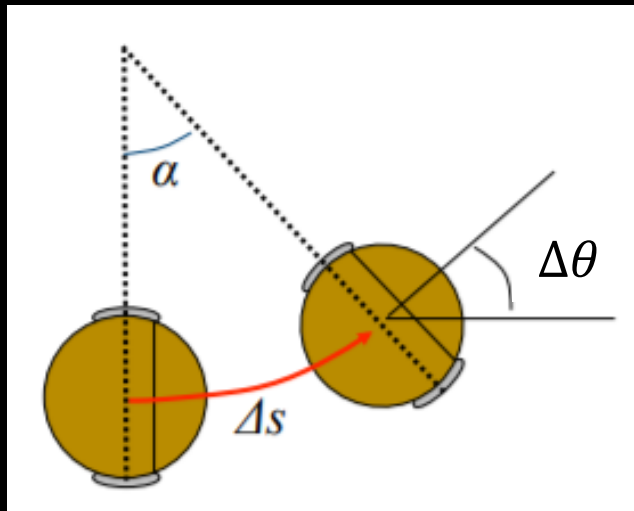
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- (2)  $\Delta s_r = (R + 2L)\alpha$
- (3)  $\Delta s = (R + L)\alpha$
- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$

• Use  $\alpha$  in (1) and (2):

- $\frac{\Delta s_l}{R} = \frac{\Delta s_r}{R+2L} \leftrightarrow (R + 2L)\Delta s_l = R(\Delta s_r)$
- $\leftrightarrow 2L\Delta s_l = R(\Delta s_r - \Delta s_l)$
- $\leftrightarrow R = \frac{2L\Delta s_l}{\Delta s_r - \Delta s_l}$

# Modeling Motion

- Start at pose  $X_{t-1}$ , move right/left wheel by  $\Delta s_r$  and  $\Delta s_l$ , what is pose  $X_t$ ?
- Model the change in angle  $\Delta\theta$  and the distance travelled  $\Delta s$ 
  - (assume that the robot is travelling on a circular arc of constant radius)



For circular arcs:

- (1)  $\Delta s_l = R\alpha$
- (2)  $\Delta s_r = (R + 2L)\alpha$
- (3)  $\Delta s = (R + L)\alpha$
- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- (5)  $R = \frac{2L\Delta s_l}{\Delta s_r - \Delta s_l}$
- Use (5) in (1):
  - $\alpha = \frac{\Delta s_l}{R} = \frac{\Delta s_l(\Delta s_r - \Delta s_l)}{2L\Delta s_l} = \frac{\Delta s_r - \Delta s_l}{2L} = \Delta\theta$

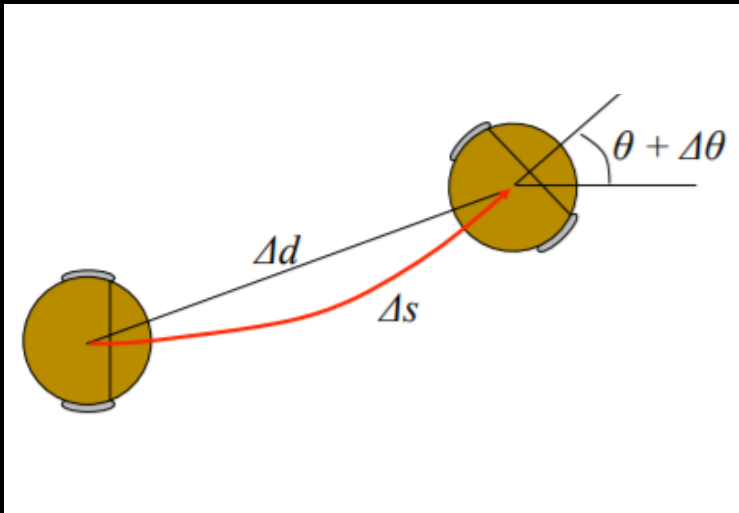
# Modeling Motion

- Start at pose  $X_{t-1}$ , move right/left wheel by  $\Delta s_r$  and  $\Delta s_l$ , what is pose  $X_t$ ?
- Model the change in angle  $\Delta\theta$  and the distance travelled  $\Delta s$ 
  - (assume that the robot is travelling on a circular arc of constant radius)
  - (assume that the motion is small,  $\Delta d \approx \Delta s$ )

For circular arcs:

- (1)  $\Delta s_l = R\alpha$
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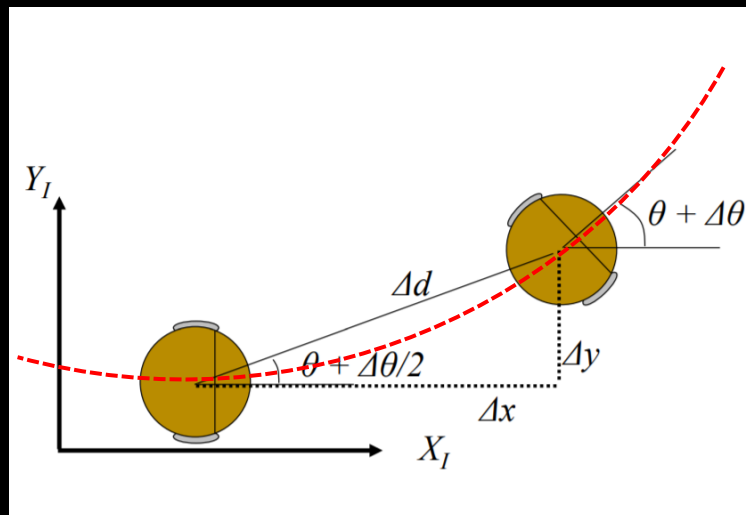
- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- (5)  $R = \frac{2L\Delta s_l}{\Delta s_r - \Delta s_l}$
- (6)  $\Delta\theta = \frac{\Delta s_r - \Delta s_l}{2L}$



# Modeling Motion

- Start at pose  $X_{t-1}$ , move right/left wheel by  $\Delta s_r$  and  $\Delta s_l$ , what is pose  $X_t$ ?
- Model the change in angle  $\Delta\theta$  and the distance travelled  $\Delta s$ 
  - (assume that the robot is travelling on a circular arc of constant radius)
  - (assume that the motion is small,  $\Delta d \approx \Delta s$ )

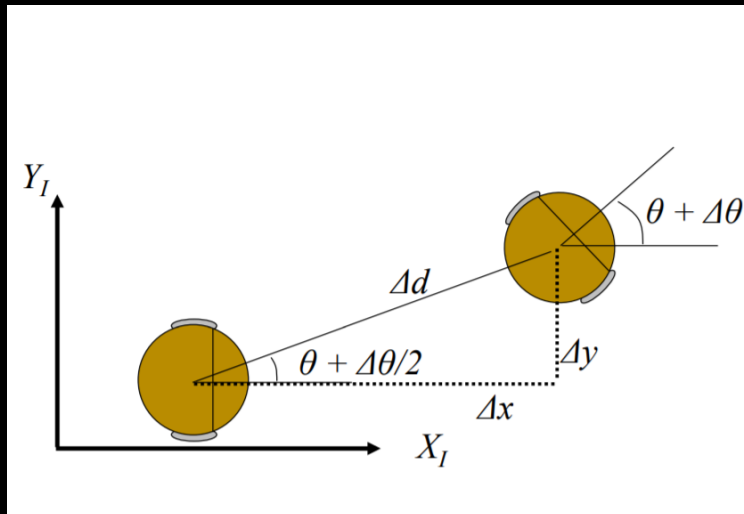
For circular arcs:



- (1)  $\Delta s_l = R\alpha$
- (2)  $\Delta s_r = (R + 2L)\alpha$
- (3)  $\Delta s = (R + L)\alpha$
- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- (5)  $R = \frac{2L\Delta s_l}{\Delta s_r - \Delta s_l}$
- (6)  $\Delta\theta = \frac{\Delta s_r - \Delta s_l}{2L}$
- (7)  $\Delta x = \Delta s \cos(\theta + \Delta\theta/2)$
- (8)  $\Delta y = \Delta s \sin(\theta + \Delta\theta/2)$

# Modeling Motion

- Start at pose  $X_{t-1}$ , move right/left wheel by  $\Delta s_r$  and  $\Delta s_l$ , what is pose  $X_t$ ?
- Model the change in angle  $\Delta\theta$  and the distance travelled  $\Delta s$ 
  - (assume that the robot is travelling on a circular arc of constant radius)
  - (assume that the motion is small,  $\Delta d \approx \Delta s$ )



- (4)  $\Delta s = \frac{\Delta s_l + \Delta s_r}{2}$
- (6)  $\Delta\theta = \frac{\Delta s_r - \Delta s_l}{2L}$
- (7)  $\Delta x = \Delta s \cos(\theta + \Delta\theta/2)$
- (8)  $\Delta y = \Delta s \sin(\theta + \Delta\theta/2)$
- $X_t = f(x, y, \theta, \Delta s_r, \Delta s_l)$

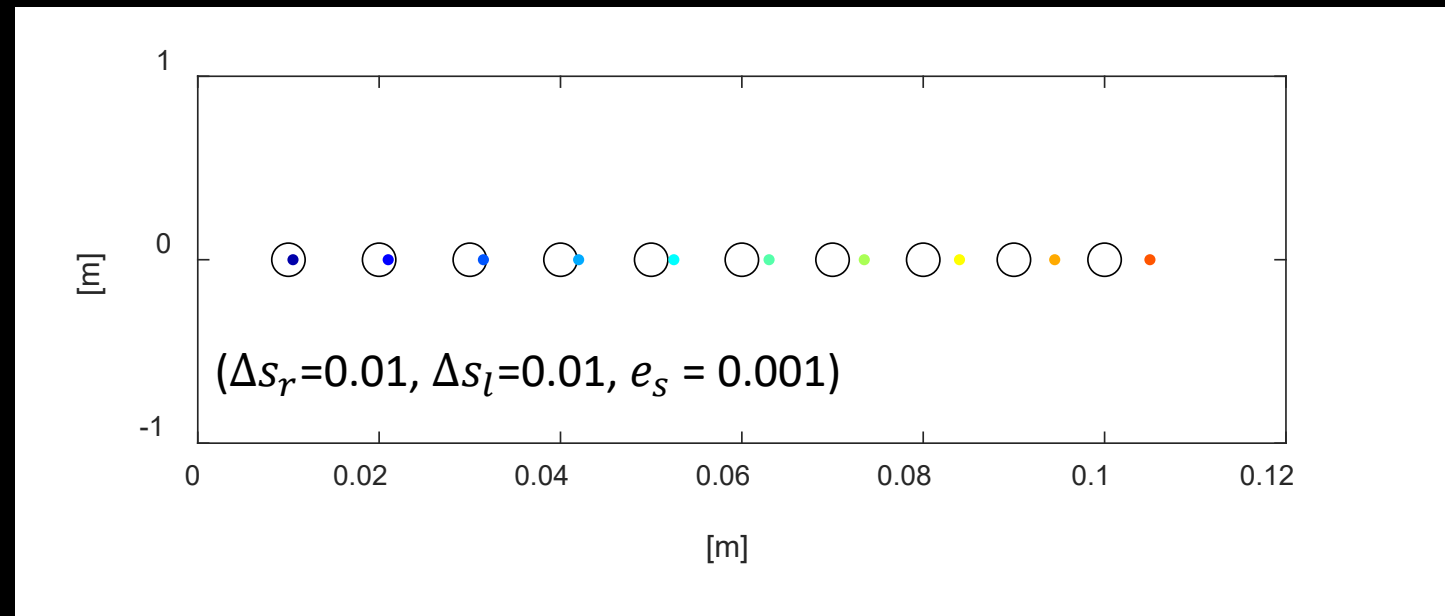
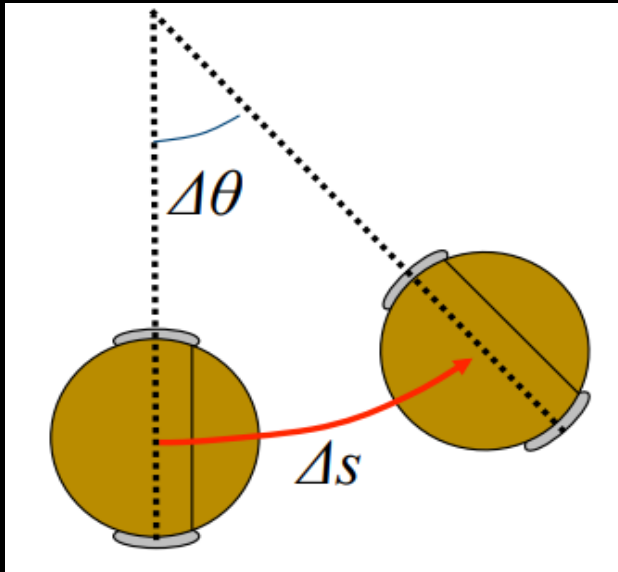
- $X_t = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{bmatrix}$

$$= \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_l + \Delta s_r}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right) \\ \frac{\Delta s_l + \Delta s_r}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right) \\ \frac{\Delta s_r - \Delta s_l}{2L} \end{bmatrix}$$

# Modeling Motion

- How do wheel rotation errors propagate into positioning errors?

- $\Delta s = d + e_s$ 
  - $\Delta x = \frac{\Delta s_l + \Delta s_r + e_s}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right)$
  - $\Delta y = \frac{\Delta s_l + \Delta s_r + e_s}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right)$

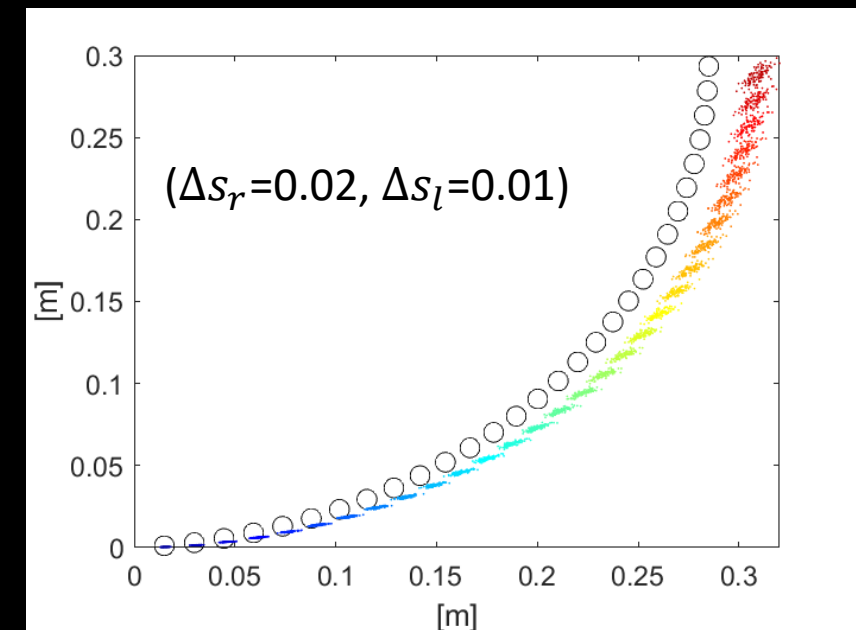
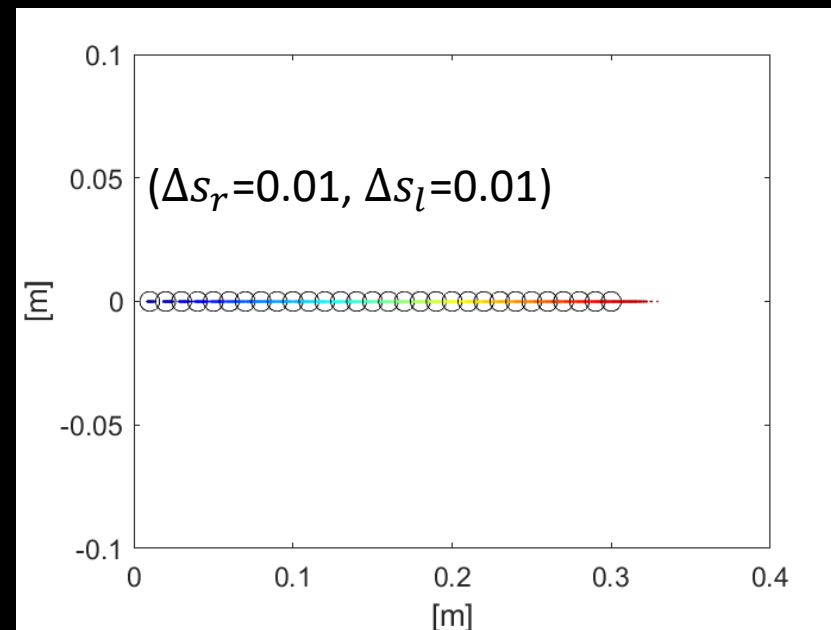
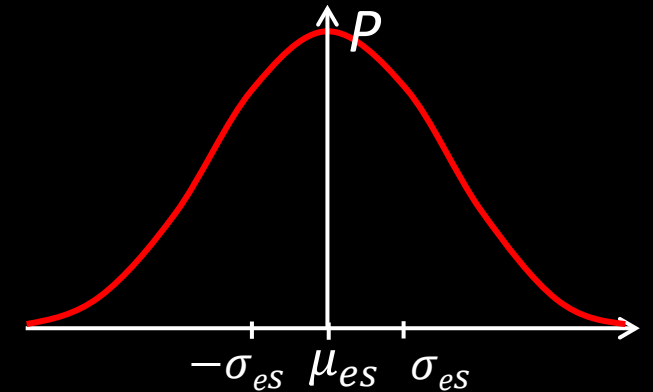
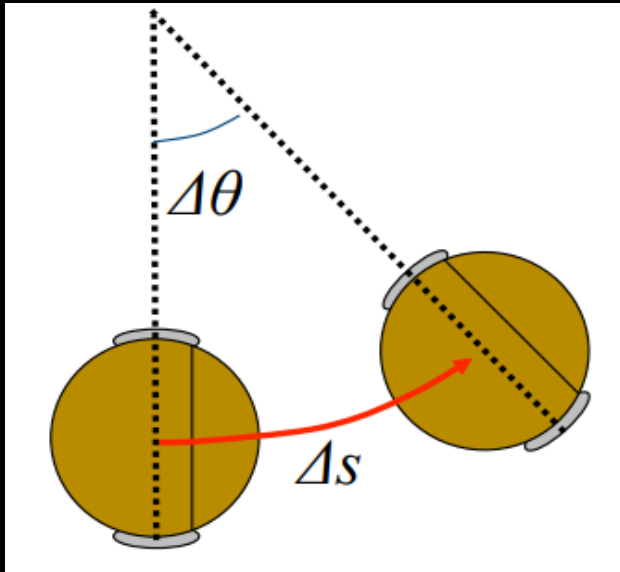




# Modeling Motion

- How do wheel rotation errors propagate into positioning errors?

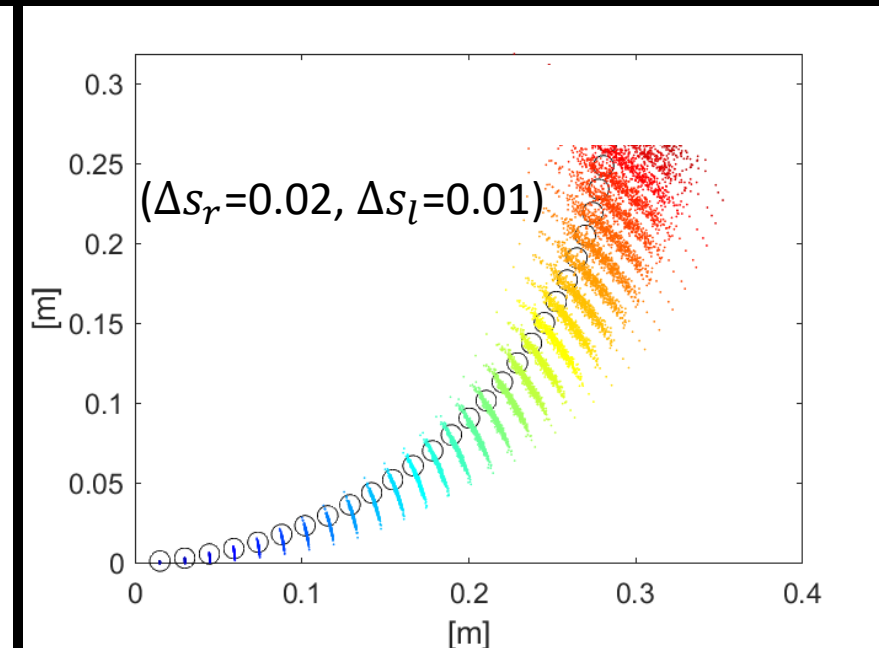
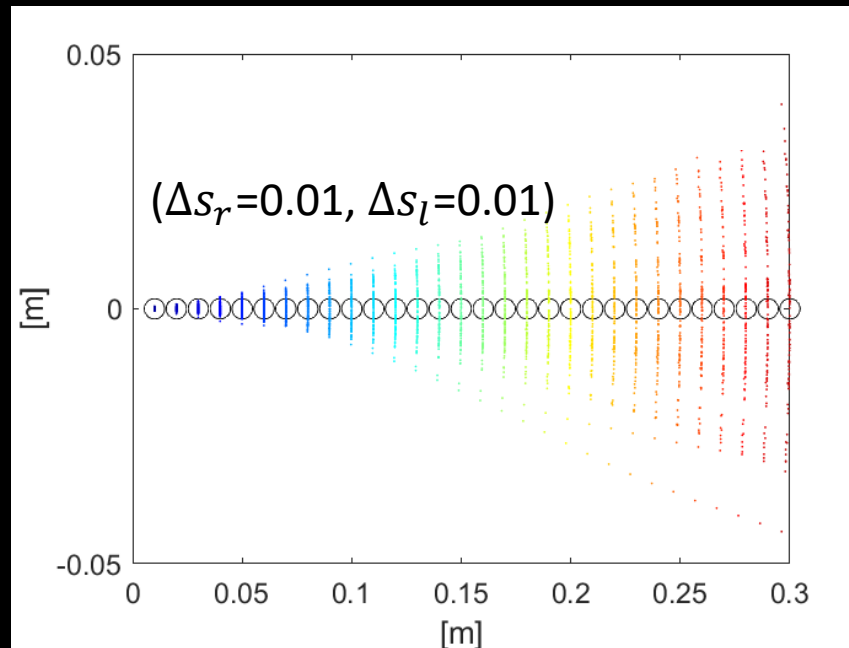
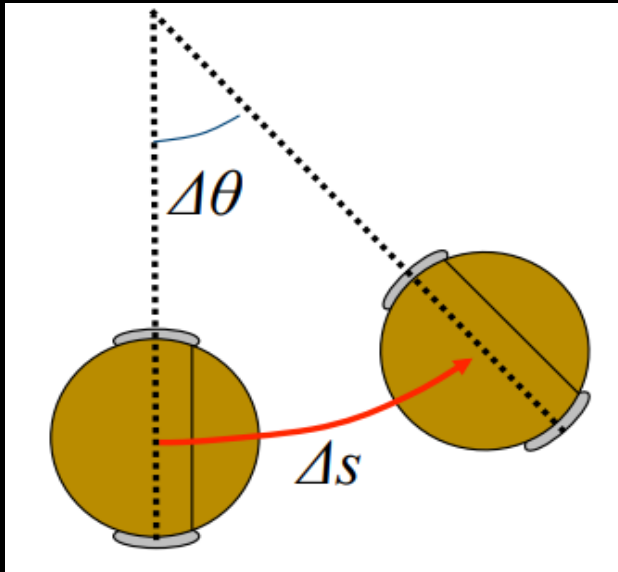
- $\Delta s = d + e_s$ 
  - $\Delta x = \frac{\Delta s_l + \Delta s_r + e_s}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right)$
  - $\Delta y = \frac{\Delta s_l + \Delta s_r + e_s}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{4L}\right)$
  - $e_s$  ( $\mu_{e_s} = 1\text{mm}$ ,  $\sigma_{e_s} = 2\text{mm}$ )



# Modeling Motion

- How do wheel rotation errors propagate into positioning errors?

- $\Delta\theta = \beta + e_\theta, e_\theta = 1^\circ$ 
  - $\Delta x = \Delta s \cos\left(\theta + \frac{\beta}{2} + e_\theta\right) = 0.1000m$
  - $\Delta y = \Delta s \sin\left(\theta + \frac{\beta + e_\theta}{2} + e_\theta\right) = 0.0175m$
  - $e_\theta (\mu_{e_\theta} = 0^\circ, \sigma_{e_\theta} = 1^\circ)$



## Sources and References

- <http://www.cs.cmu.edu/~rasc/Download/AMRobots4.pdf>
- [https://www.ti.com/lit/ug/sbau305b/sbau305b.pdf?ts=1599417595209&ref\\_url=https%253A%252F%252Fwww.google.com%252F](https://www.ti.com/lit/ug/sbau305b/sbau305b.pdf?ts=1599417595209&ref_url=https%253A%252F%252Fwww.google.com%252F)
- <https://hmc.edu/lair/ARW/ARW-Lecture01-Odometry.pdf>
- Matlab Tech Talks on Sensor Fusion (<https://www.youtube.com/watch?v=6qV3YjFppuc>)

## Introduction to Lab 2

- Last 10min of class

## Lab 2 Bluetooth

- *NB: The lab has changed slightly from last year!*
- Good example from last year
  - Owen Deng: <https://qd39l.github.io/fast-robots/lab2.html>
- Lab 2
  - <https://cei-lab.github.io/FastRobots-2023/Lab2.html>
  - Prelab (setup VM/Jupyter and Artemis, read through the code base)
  - Tasks
    - Change the MAC address/UUID
    - Send/receive ECHO
    - Get\_Time\_Millis()
    - Notification handler
    - Get\_Temp\_5s (T:1500|C:24|T:2657|C:24.5 ...etc)
    - Get\_Temp\_Rapid
    - Consider Artemis storage